# COSC 030, Fall 2016: Problem Set $\#4$

**Assigned:** Thursday, 9/29.
**Due:** Tuesday, 10/11, at the beginning of class (hand in hard copy).
**Lectures Covered:** Weeks 5 and 6.
**Academic Integrity:** You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

## Problems

1. Prove using induction that for every integer $n \geq 1$: $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$.

2. Let $P(n)$ = "$n = 3n$". What's wrong with the following inductive proof for the claim that $P(n)$ holds for every $n \in \mathbb{N}$?

   - **Base Case** $(n = 0)$**:** $P(0)$ claims that $0 = 3 \cdot 0$, which is clearly true.
   - **Step:** Assume $P(n)$. We will show $P(n+1)$.
     (a) $n = 3 \cdot n$ (Inductive Hypothesis)
     (b) $n + 1 = 3 \cdot n + 1$ (algebra: adding the same value—in this case, 1—to both sides of an equality maintains the equality).
     (c) $P(n+1)$ (follows from $b$ which is the same as the definition of $P(n+1)$)

3. Prove using *strong* induction that every integer greater than $0$ can be written as the sum of distinct powers of 2.

   (*Note:* A "power of 2" is a value that can be written as $2^i$ for some $i \in \mathbb{N}$. By "distinct" in this context I mean that you cannot use the same power of 2 more than once in the sum for a given a value. For example, to show the statement holds for 7, I could note that $7 = 2^2 + 2^1 + 2^0$—that is, 7 is shown here to equal the sum of three distinct powers of 2.)

   (*Hint:* In your proof, feel free to use the following mathematical fact: for any integer $x > 0$, there exists an $i \in \mathbb{N}$ and $k \in \mathbb{N}$, such that $x = 2^i + k$ and $k < 2^i$.)

4. Prove using induction that for every postal cost of at least 12 cents it is possible to form that cost using combinations of $4$ and $5$ cent stamps. It is up to you whether you use normal or strong induction, but clearly label what you decide.

   (*Hint:* You might find it useful to prove an extended base case that covers $P(b)$, $P(b+1)$, $P(b+2)$, and $P(b+3)$ before continuing to the inductive step.)