Problems

1. A prime number is whole number greater than 1 that is only divisible by 1 and itself (e.g., 2, 3, 5, 7, 11, 13, 17...). A useful property of prime numbers is that they are always odd.
   - Prove this property using a proof by contraposition.
   - Prove this property using a proof by contradiction.

2. Let $A = \{a, b, c\}$, $B = \{\text{dog, cat, house, a}\}$, and $C = \{a, \text{cat}\}$. Specify whether each of the following statements is true or false.
   (a) $C \subset B$
   (b) $A \subseteq B$
   (c) $C \subseteq C$
   (d) $(\mathbb{R} - A) \cup C$ is finite.
   (e) $|A \cup C| = |B|

3. Describe using set builder notation a set $D$ such that: (a) $D$ is infinite; and (b) $|B - D| = 1$ (where $B$ is defined in the preceding problem).

4. Define a function $f$ that satisfies the following three properties: (1) its domain and range both contain 3 elements; (2) it is one-to-one; and (3) it is not onto.

5. Let $\mathbb{D}_E$ be the set of all words in the standard English dictionary. Let $h : \mathbb{D}_E \rightarrow \{a, b, c, ..., y, z\}$ be the function that maps each word $w \in \mathbb{D}_E$ to the first letter in $w$ (e.g., $h(\text{cat}) = c$). Let $\{a_i\}$, be the sequence defined for $1 \leq i \leq 3$ where $a_1 = \text{math}$, $a_2 = \text{methods}$, $a_3 = \text{problem}$.

What is the value of the following summation:

$$\sum_{i=2}^{3} |\{h(a_i)\} \cap \{m\}|$$