COSC 030, Fall 2015: Problem Set $\#8$

Assigned: Tuesday, 11/10.
Due: Tuesday, 11/17. **Lectures Covered:** Week 11 (including the leftover material from Week 10).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. In class we considered the *Search For* 1 problem in which you are provided a sequence containing the integers from 1 to n in some order. To solve the problem, you must output the position in the sequence that contains 1. For example, if $n = 4$ and you are provided input $(2, 1, 3, 4)$, the correct response is 2, as the 1 is in the second position of the input sequence.

   Consider the simple algorithm that iterates through the sequence, checking each position in order until it finds the 1—at which point it returns that position. Describe a probabilistic input source for this problem for which the *average case* step complexity of the above algorithm is $\Theta(\sqrt{n})$.

   (*For full credit:* show the calculation of average case complexity that yields your result.)

2. For each of the following recurrences specify whether or not it is a linear homogeneous recurrence relation with constant coefficients. If it *is*, provide the degree. If it *is not*, provide an explanation for why it is not.

   (a) $a_n = (1.9)a_{n-1}$
   (b) $a_n = a_{n-1} + a_{n-2}^2$
   (c) $a_n = a_{n-1} + a_{n-3}$
   (d) $a_n = 2a_{n-1} + 1$
   (e) $a_n = 7a_{n-5}$
   (f) $a_n = 2a_n + na_{n-3}$

3. Solve the recurrence $a_n = 4a_{n-1} - 3a_{n-2}$, where $a_0 = 0$ and $a_1 = 1$.

   (*For full credit:* show your work.)

4. Using the Master Theorem: Solve the recurrence $f(n) = f(n/2) + (1/2)n^2$ (for values of n that can be expressed as a power of 2). Your answer can be in asymptotic notation.

5. Using the Master Theorem: Solve the recurrence $f(n) = 2f(n/4) + \sqrt{n}$ (for values of n that can be expressed as powers of 4). Your answer can be in asymptotic notation.