COSC 030, Fall 201: Problem Set #6

Assigned: Tuesday, 10/13.
Due: Thursday, 10/22. *Notice that this is a Thursday deadline. The problem set will cover material taught on Tuesday 10/20.*
Lectures Covered: Weeks 7 and 8 (Chapters 6.1, 6.2, 6.3).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. The National Park Service's employee intranet requires a password that is either five (lower case) letters long or the name of a state in the United States (in lower case). For state names with two words (e.g., New Jersey) the password must skip the space (e.g., newjersey).

   How many possible passwords are there?

   (Show your work and mention which rules you are applying to derive your answer.)

2. Assume you have a network router that stores each incoming packet in one of 3 queues that can each hold 12 packets. Assume at some point all 3 queues are empty when suddenly a group of n packets arrives all at once for the router to divide into its queues. You do not know in advance the strategy the router will use to decide in which queue to place each packet. What is the smallest value of n that guarantees at least one queue will be full (i.e., contain 12 packets)?

3. A group of six people walk into an art museum. Assume that if any two people from this group strike up a conversation they can determine whether or not they have the same taste in art. Prove that there must exist a subset of three people from this group such that either: (a) all pairs in this subset have the same taste in art; or (b) none of the pairs in this subset have the same taste in art. Use the pigeon hole principle in formulating your answer.

   (If you come up with a real simple answer, then make sure you understand what the problem is asking—a correct answer should require several steps of reasoning.)

4. Assume a classroom has 30 students. The professor wants to choose a group of 3 students to grade a problem set. How many possible grading groups can he choose?

5. Assume we want to form a police line-up. The line-up room has 5 positions, labelled 1 to 5. We have a pool of 10 people to use in forming our 5-person line-up. One person in the pool is the actual suspect. For a line-up to be valid it must contain the suspect and the suspect must be in position 3.

   How many different valid 5-person line-ups can we form from this pool?