COSC 030, Fall 2015: Problem Set #5

Assigned: Tuesday, 10/6.
Due: Tuesday, 10/13, at the beginning of class (hand in hard copy).
Lectures Covered: Week 6 (Chapters 5.3, 5.4).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. The product operator $\prod$ multiples values in a sequence. That is, $\prod_{k=0}^{n} a_k = a_0 \cdot a_1 \cdot ... \cdot a_n$. Provide a recursive definition for $\prod$.

2. Provide a recursive definition for the function $f(n) = n^2$.
   (Hint: You might find it useful to remember that $(n-1)^2 = n^2 - 2n + 1$.)

3. Provide a recursive definition for the set containing all positive powers of two (i.e., $\{2^i \mid i \in \mathbb{N}\}$).

4. In class we studied the set S recursively defined as follows:

   - *Basis:* $3 \in S$.
   - *Recursive Step:* If $x \in S$ and $y \in S$ then $x + y \in S$.

   Use *structural induction* to prove that every $x \in S$ is a multiple of 3.

5. In class, we studied the following recursive algorithm which calculates factorials
   (Recall: for $n > 0$, $n! = n \cdot (n-1) \cdot ... \cdot 2 \cdot 1$, and $0! = 1$):

   Factorial($n \in \mathbb{N}$)
   if $n = 0$ **then return** 1
   else return $n \cdot$ **Factorial**$(n-1)$

   Using induction, prove that **Factorial**(n) returns $n!$.

6. Using your answer from problem 2, describe a recursive algorithm that takes a natural number as input and then returns its square.