COSC 030, Fall 2015: Problem Set $\#4$

Assigned: Tuesday, 9/29.
Due: Tuesday, 10/6, at the beginning of class (hand in hard copy).
Lectures Covered: Week 5.
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Consider the following algorithm:

   RepeatMultiply$(a, b \in \mathbb{N})$
   $result \leftarrow 1$
   for $i \leftarrow 1$ to b
   $result \leftarrow result \times a$
   return $result$

   Prove using induction that for any $a, b \in \mathbb{N}$, after every step of **RepeatMultiply**(a, b): $result \geq 0$. In more detail, prove using induction that the proposition $P(n) =$ "$result \geq 0$ after n steps of the algorithm" holds for all $n \geq 1$.

2. Prove using induction that for every integer $n \geq 1$: $\sum_{i=1}^{n} 2^i = 2^{n+1} - 2$.

3. Let $P(n) =$ "$n = 3n$". What's wrong with the following inductive proof for the claim that $P(n)$ holds for every $n \in \mathbb{N}$?

   - **Base Case** $(n = 0)$**:** $P(0)$ claims that $0 = 3 \cdot 0$, which is clearly true.
   - **Step:** Assume $P(n)$. We will show $P(n + 1)$.
     (a) $n = 3 \cdot n$ (Inductive Hypothesis)
     (b) $n + 1 = 3 \cdot n + 1$ (algebra: adding the same value—in this case, 1—to both sides of an equality maintains the equality).
     (c) $P(n + 1)$ (follows from b which is the same as the definition of $P(n + 1)$)

4. Prove using *strong* induction that every integer greater than 0 can be written as the sum of distinct powers of 2.

   (*Note:* A "power of 2" is a value that can be written as 2^i for some $i \in \mathbb{N}$. By "distinct" in this context I mean that you cannot use the same power of 2 more than once in the sum for a given a value. For example, to show the statement holds for 7, I could note that $7 = 2^2 + 2^1 + 2^0$—that is, 7 is shown here to equal the sum of three distinct powers of 2.)

   (*Hint:* In your proof, feel free to use the following mathematical fact: for any integer $x > 0$, there exists an $i \in \mathbb{N}$ and $k \in \mathbb{N}$, such that $x = 2^i + k$ and $k < 2^i$.)