COSC 030, Fall 2014: Problem Set #9

Assigned: Tuesday, 11/11.
Due: Tuesday, 11/18. **Lectures Covered:** Week 12 (Chapters 10, 1, 10.2, and 10.3).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Draw each of the following graphs:
   - K_4
   - $K_{9,2}$
   - W_3
   - Q_2
2. Draw an undirected *pseudograph* that has 3 vertices labelled a, b, c, and 6 edges. Describe the graph mathematically (i.e., define V and E).
3. How many different undirected pseudographs could you have drawn in answering the previous problem? (Assume you must always use the vertex labels a, b, and c.)
4. Assume that I have a bag that contains all simple undirected graphs containing n vertices that are labelled $1, 2, ..., n$, and that I reach into this bag and pull out a graph (every graph is equally likely to be selected).
   (a) For $n = 3$: what is the probability that I pull out K_3?
   (b) For general $n > 1$: what is the probability that I pull out K_n? (Express this probability as a function on n. This function can only include basic arithmetic operations: addition, subtraction, division, multiplication, and exponentiation [e.g., x^y].)
5. Prove the Handshaking Theorem we discussed in class *using induction*.
6. For a given $m > 1$, what is the size of the maximum matching in the complete bipartite graph $K_{m,m}$?
7. Use the pigeonhole principle to prove that your answer from the previous problem is correct (i.e., that there cannot be a matching larger than the size given as your answer.)