COSC 030, Fall 2014: Problem Set $\#8$

Assigned: Tuesday, 11/4.
Due: Tuesday, 11/11. **Lectures Covered:** Week 11 (including the leftover material from Week 10).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Note: *The problem set continues onto a second page. Do not skip the two problems on the second page.*

Problems

1. In class we considered the *search for* 1 problem in which you are provided a sequence containing the integers from 1 to n in some order. To solve the problem, you must output the position in the sequence that contains 1. (For example, if $n = 3$ and you are provided input $2, 1, 3$, the correct response is 2 as the 1 is in the second position of the input sequence).

   Consider the simple algorithm that iterates through the sequence, checking position 1 (*if* $a_1 = 1$...), then 2, and so on, until it finds the 1 (at which point it returns the position).

   Assume the input to the algorithm is generated as follows: A coin is flipped 3 times. If all 3 flips come up heads, the input is sorted into decreasing order (i.e., $n, n-1, ..., 2, 1$), else it is sorted into increasing order (e.g., $1, 2, ..., n-1, n$).

   What is the average case step complexity of this algorithm on this input source?

   (To receive credit show your work.)

2. Describe an input source for the algorithm in the preceding problem that will provide an average case step complexity that is *not* $\Theta(n)$ and is *not* $\Theta(1)$. Justify your answer by calculating the average case step complexity of this new input source. (To receive credit show your work.)

3. For each of the following recurrences specify whether or not it is a linear homogeneous recurrence relation with constant coefficients. If it *is*, provide the degree. If it *is not*, provide an explanation for why it is not.

   (a) $a_n = (1.9)a_{n-1}$
   (b) $a_n = a_{n-1} + a_{n-2}^2$
   (c) $a_n = a_{n-1} + a_{n-3}$
   (d) $a_n = 2a_{n-1} + 1$
   (e) $a_n = 7a_{n-5}$
   (f) $a_n = 2a_n + na_{n-3}$

4. Solve the recurrence $a_n = 4a_{n-1} - 3a_{n-2}$, where $a_0 = 0$ and $a_1 = 1$.

   (To receive credit show your work.)

5. Solve the recurrence $f(n) = f(n/2) + (1/2)n^2$ (for values of n that can be expressed as a power of 2). Your answer can be in asymptotic notation.

6. Solve the recurrence $f(n) = 2f(n/4) + \sqrt{n}$ (for values of n that can be expressed as powers of 4). Your answer can be in asymptotic notation.