COSC 030, Fall 2014: Problem Set $\#5$

Assigned: Tuesday, 9/30.
Due: Tuesday, 10/7, at the beginning of class (hand in hard copy).
Lectures Covered: Week 6 (Chapters 5.3, 5.4).
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Provide a recursive definition for the function $f(n) = n^2$.
   (Hint: You might find it useful to remember that $(n-1)^2 = n^2 - 2n + 1$.)

2. Provide a recursive definition for the set that contains all strings that are formed only from letters that appear in the first name of a Beatle.

3. In class we studied the set S recursively defined as follows:

   - *Basis:* $3 \in S$.
   - *Recursive Step:* If $x \in S$ and $y \in S$ then $x + y \in S$.

   Use *structural induction* to prove that every $x \in S$ is a multiple of 3.

4. In class, we studied the following recursive algorithm which calculates factorials
   (Recall: for $n > 0$, $n! = n \cdot (n-1) \cdot ... \cdot 2 \cdot 1$, and $0! = 1$):

   Factorial($n \in \mathbb{N}$)
   if $n = 0$ **then return** 1
   else return $n\cdot$ **Factorial**($n - 1$)

   Using induction, prove that **Factorial**(n) returns $n!$.

5. Using your answer from problem 1, describe a recursive algorithm that takes a natural number as input and then returns its square.

6. Provide an iterative algorithm that solves the same square problem as above. Which algorithm would you rather use in a real system and why?