Assignments: Tuesday, 9/16.
Due: Tuesday, 9/23, at the beginning of class (hand in hard copy).
Lectures Covered: Week 4.
Academic Integrity: You must work alone on the problem set and not consult outside sources (with the exception of the professor and teaching assistants). See the syllabus for details on the academic integrity policy for problem sets.

Problems

1. Consider the Two Search problem which provides an algorithm a sequence \( a_1, a_2, \ldots, a_n \) of natural numbers as input, and requires the algorithm to return \( true \) if at least one \( a_i \) value equals 2, and otherwise requires the algorithm to return \( false \).

Why is the following solution to the Two Search problem not a valid computer algorithm?

\[
\text{SplitSearch}(a_1, a_2, \ldots, a_n) \\
x \leftarrow \text{Contains2}(a_1, \ldots, a_{\lfloor n/2 \rfloor}) \\
y \leftarrow \text{Contains2}(a_{\lfloor n/2 \rfloor}+1, \ldots, a_n) \\
\text{if } x = true \text{ or } y = true \text{ then return } true \\
\text{else return } false
\]

2. Describe a correct algorithm for the Two Search problem.

3. What is the exact (i.e., not asymptotic) worst-case step complexity of your algorithm? Provide an explanation for how you arrived at this value.

4. For each of the following statements, specify if it is \( true \) or \( false \).
   \begin{enumerate}
   \item \( 19n^2 + 1000n^3 \) is \( \Theta(n^2) \)
   \item \( 2n + 20 \) is \( O(n^2) \)
   \item \( 2^{\log_2 n + 1} \) is \( \Omega(n) \)
   \item \( 2^n \) is \( O(n^2) \)
   \item \( \sqrt{n} \) is \( \Omega((\log n)^2) \)
   \end{enumerate}

5. Consider the function \( f(x) = 25x \).
   \begin{enumerate}
   \item Define a function \( g(x) \) such that \( g(x) \) is \( O(f(x)) \) but \( g(x) \) is not \( \Omega(f(x)) \).
   \item Define a function \( h(x) \) such that \( h(x) \) is \( \Omega(f(x)) \) but \( h(x) \) is not \( O(f(x)) \).
   \item Define a function \( j(x) \) that is different than \( f(x) \) but is also \( \Theta(f(x)) \).
   \end{enumerate}