COSC545: Homework 1

The following problems are from Sipser.

**Problem 1.** (50 points.) Consider the problem of testing whether a Turing Machine $M$ on input $w$ ever moves its head to the left when its tape-head is on the left-most cell. (Recall that if a Turing Machine moves its tape-head left in this case, its head just stays in the same place.) Formulate this problem as a language and show that it is undecidable.

**Problem 2.** (50 points.) Consider the problem of testing whether a Turing Machine $M$ on input $w$ ever moves its head to the left at *any* point during its computation. Formulate this problem as a language and show that it is decidable.

**Problem 3.** (50 points.) Show that there exists a language that cannot be decided by any Turing Machine that has an oracle for the halting problem. (Both define the language, and then prove that there is no such Turing Machine that decides it.)