Parsing with Context Free Grammars

CMSC 723 / LING 723 / INST 725

MARINE CARPUAT
marine@cs.umd.edu

Today's Agenda

- Grammar-based parsing with CFGs
  - CKY algorithm
- Dealing with ambiguity
  - Probabilistic CFGs
- Strategies for improvement
  - Rule rewriting / Lexicalization

Sample Grammar

Grammar	Lexicon
$S \rightarrow NP\ VP$	$Det \rightarrow that \mid this \mid a$
$S \rightarrow Aux\ NP\ VP$	$Noun \rightarrow book \mid flight \mid meal \mid money$
$S \rightarrow VP$	$Verb \rightarrow book \mid include \mid prefer$
$NP \rightarrow Pronoun$	$Pronoun \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$Proper\text{-}Noun \rightarrow Houston \mid NWA$
$NP \rightarrow Det\ Nominal$	$Aux \rightarrow does$
$Nominal \rightarrow Noun$	$Preposition \rightarrow from \mid to \mid on \mid near \mid through$
$Nominal \rightarrow Nominal\ Noun$	
$Nominal \rightarrow Nominal\ PP$	
$VP \rightarrow Verb$	
$VP \rightarrow Verb\ NP$	
$VP \rightarrow Verb\ NP\ PP$	
$VP \rightarrow Verb\ PP$	
$VP \rightarrow VP\ PP$	
$PP \rightarrow Preposition\ NP$	

GRAMMAR-BASED PARSING: CKY

Grammar-based Parsing

- Problem setup
  - Input: string **and a CFG**
  - Output: parse tree assigning proper structure to input string

- "Proper structure"
  - Tree that covers all and only words in the input
  - Tree is rooted at an S
  - Derivations obey rules of the grammar
  - Usually, more than one parse tree…

Parsing Algorithms

- Two basic (= bad) algorithms:
  - Top-down search
  - Bottom-up search
- A “real” algorithm:
  - CKY parsing

Top-Down Search

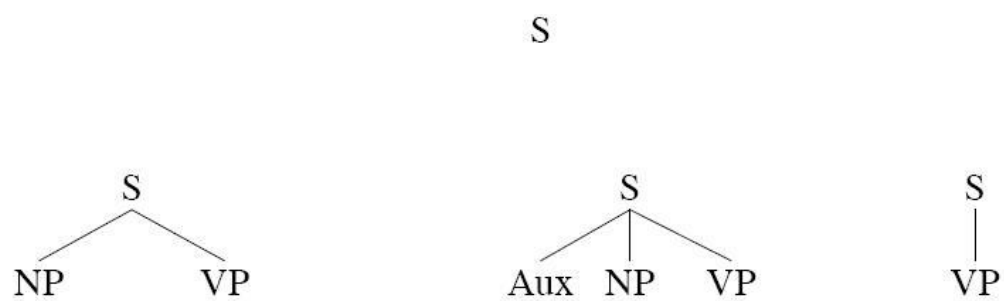
- Observation
  – trees must be rooted with an S node

- Parsing strategy
  – Start at top with an S node
  – Apply rules to build out trees
  – Work down toward leaves

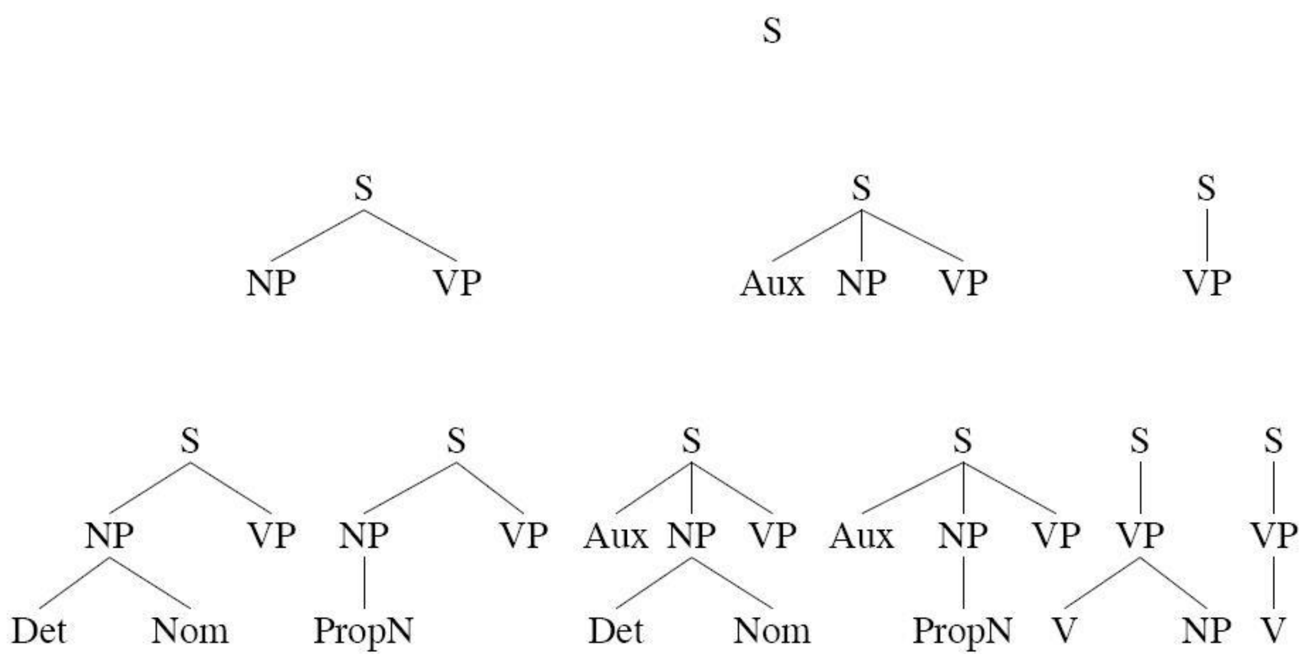
Top-Down Search

S

Top-Down Search



Top-Down Search



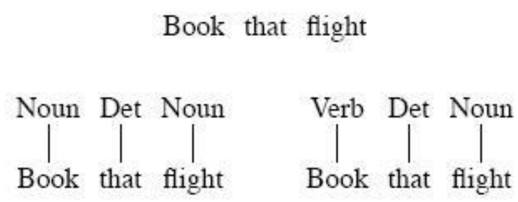
Bottom-Up Search

- Observation
  - trees must cover all input words
- Parsing strategy
  - Start at the bottom with input words
  - Build structure based on grammar
  - Work up towards the root S

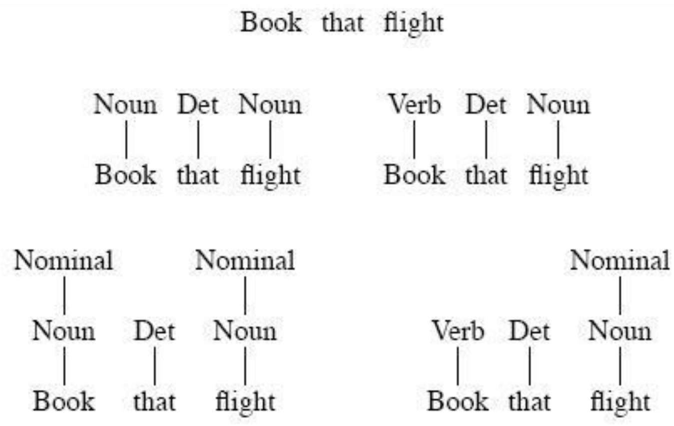
Bottom-Up Search

Book that flight

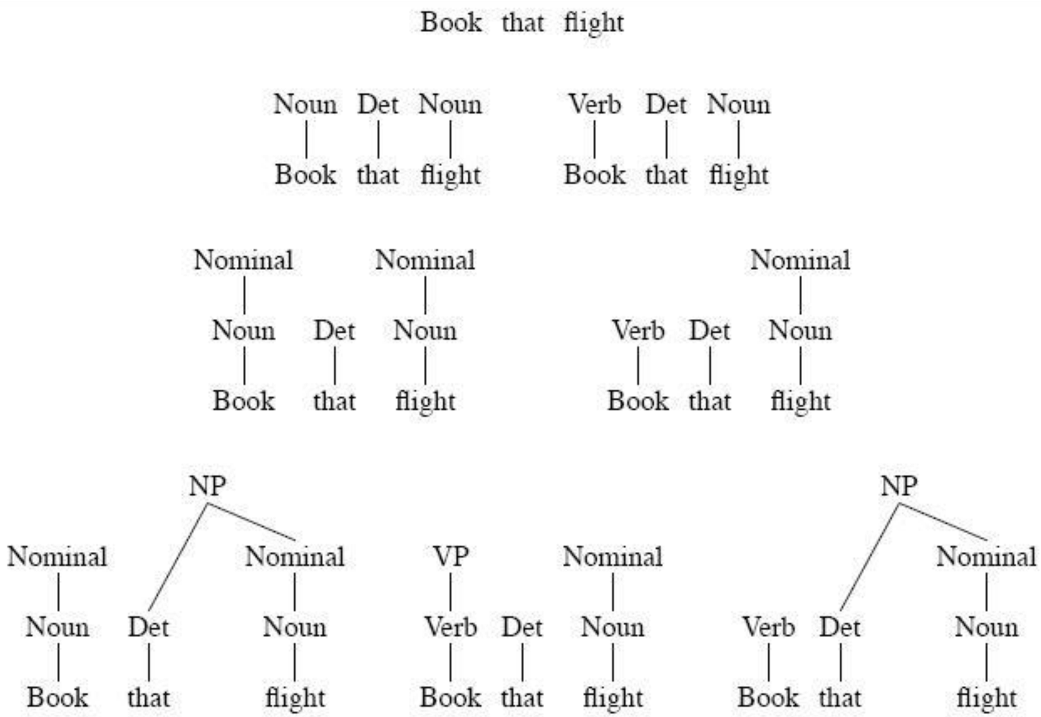
Bottom-Up Search



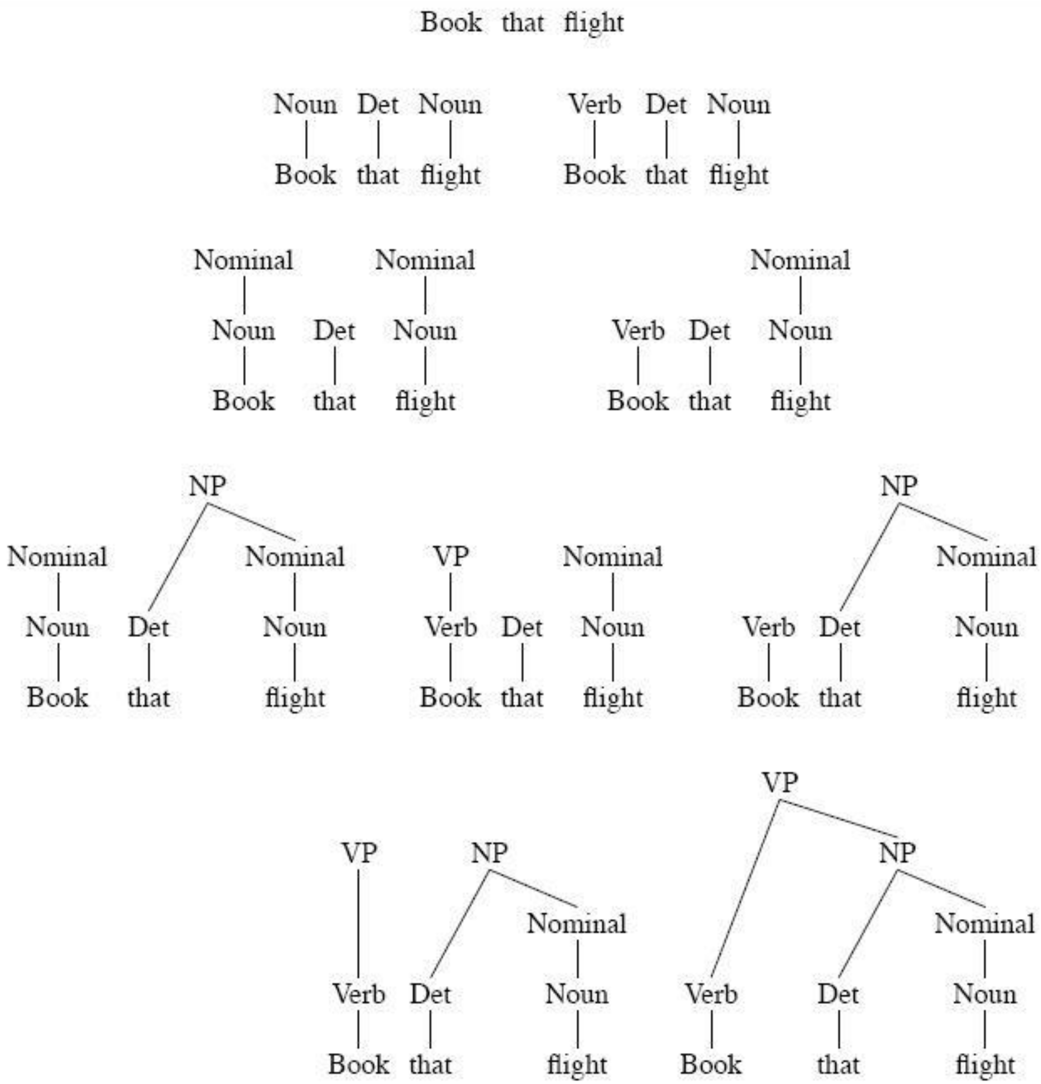
Bottom-Up Search



Bottom-Up Search



Bottom-Up Search



Top-Down vs. Bottom-Up

- Top-down search
  - Only searches valid trees
  - But, considers trees that are not consistent with any of the words
- Bottom-up search
  - Only builds trees consistent with the input
  - But, considers trees that don't lead anywhere

Parsing as Search

- Search involves controlling choices in the search space
  - Which node to focus on in building structure
  - Which grammar rule to apply

- General strategy: backtracking
  - Make a choice, if it works out then fine
  - If not, back up and make a different choice

Shared Sub-Problems

- Observation
  - ambiguous parses still share sub-trees
- We don’t want to redo work that’s already been done
- Unfortunately, naïve backtracking leads to duplicate work

Efficient Parsing with the CKY Algorithm

- Solution: Dynamic programming
- Intuition: store partial results in tables
  - Thus avoid repeated work on shared sub-problems
  - Thus efficiently store ambiguous structures with shared sub-parts
- We’ll cover one example
  - CKY: roughly, bottom-up

CKY Parsing: CNF

- CKY parsing requires that the grammar consist of binary rules in Chomsky Normal Form
  - All rules of the form:

$$A \rightarrow B\ C$$
$$D \rightarrow w$$

  - What does the tree look like?

CKY Parsing with Arbitrary CFGs

- What if my grammar has rules like
  VP → NP PP PP
  - Problem: can't apply CKY!
  - Solution: rewrite grammar into CNF
    - Introduce new intermediate non-terminals into the grammar

$A \rightarrow B\ C\ D$ ⇒ $A \rightarrow X\ D$, $X \rightarrow B\ C$

(Where X is a symbol that doesn't occur anywhere else in the grammar)

Sample Grammar

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$VP \rightarrow Verb\ NP$	
$VP \rightarrow Verb\ NP\ PP$	
$VP \rightarrow Verb\ PP$	
$VP \rightarrow VP\ PP$	
$PP \rightarrow Preposition\ NP$	

CNF Conversion

Original Grammar	CNF Version
$S \rightarrow NP\ VP$	
$S \rightarrow Aux\ NP\ VP$	
$S \rightarrow VP$	
$NP \rightarrow Pronoun$	
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CNF Conversion

Original Grammar	CNF Version
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
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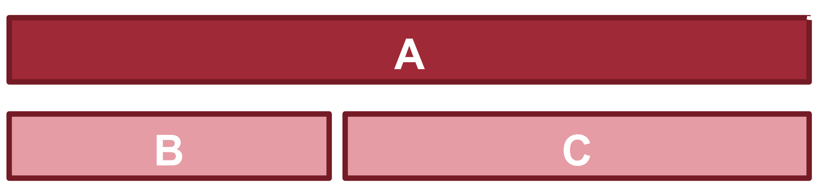
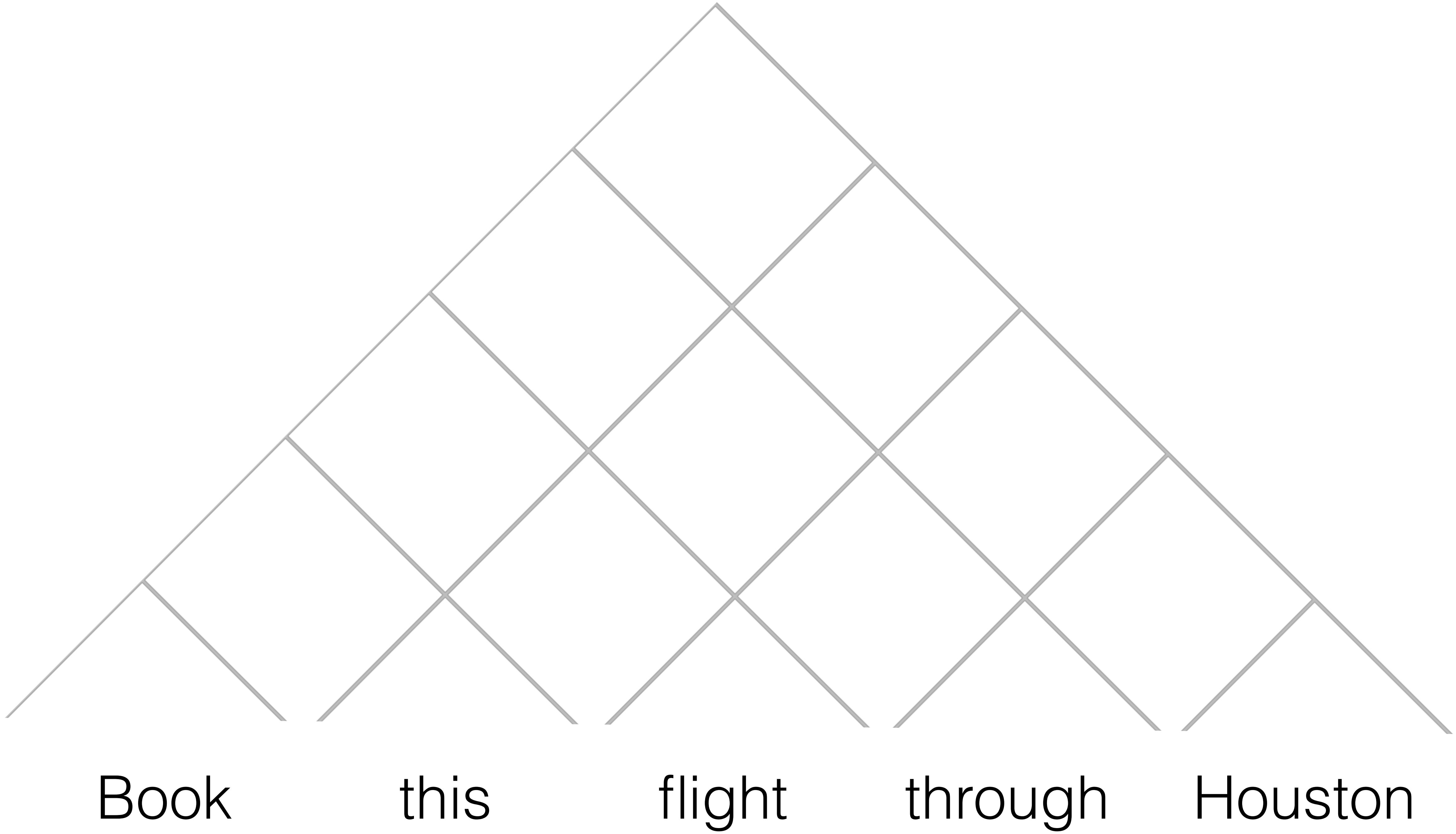
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

CKY Parsing: Intuition

- Consider the rule D → w
  - Terminal (word) forms a constituent
  - Trivial to apply
- Consider the rule A → B C
  - "If there is an A somewhere in the input, then there must be a B followed by a C in the input"
  - First, precisely define span [i, j]
  - If A spans from i to j in the input then there must be some k such that $i<k<j$
  - Easy to apply: we just need to try different values for k



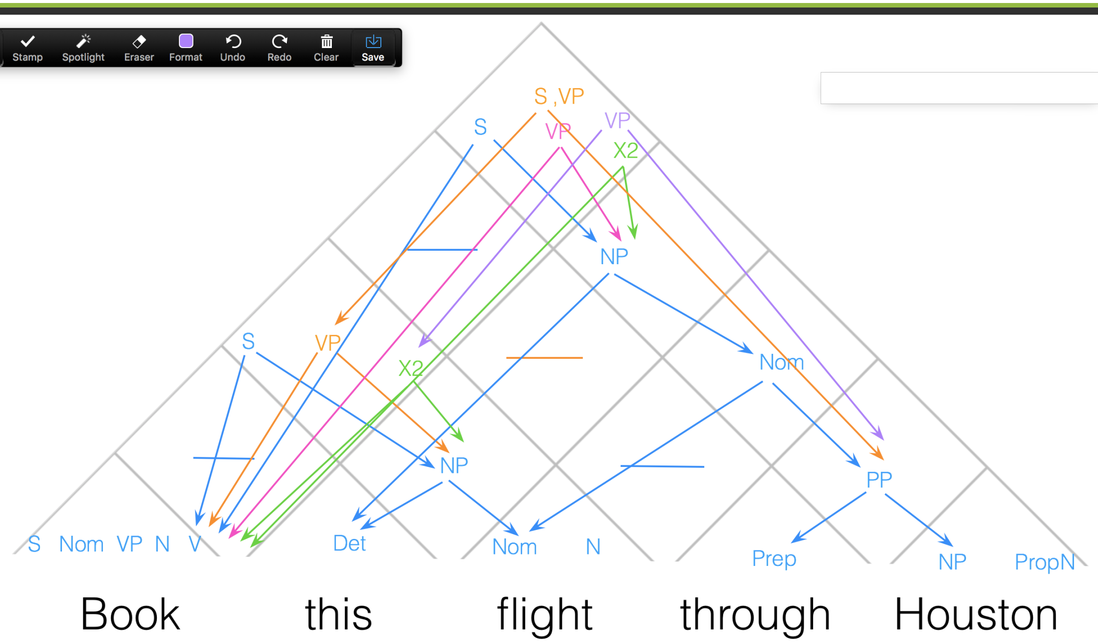


CNF (binarized) grammar

S → *NP VP*
S → *X1 VP*
X1 → *Aux NP*
S → *book* | *include* | *prefer*
S → *Verb NP*
S → *X2 PP*
S → *Verb PP*
S → *VP PP*
NP → *I* | *she* | *me*
NP → *TWA* | *Houston*
NP → *Det Nominal*

Nominal → *book* | *flight* | *meal* | *money*
Nominal → *Nominal Noun*
Nominal → *Nominal PP*
VP → *book* | *include* | *prefer*
VP → *Verb NP*
VP → *X2 PP*
X2 → *Verb NP*
VP → *Verb PP*
VP → *VP PP*
PP → *Preposition NP*

Lexicon
Det → *that* | *this* | *a*
Noun → *book* | *flight* | *meal* | *money*
Verb → *book* | *include* | *prefer*
Pronoun → *I* | *she* | *me*
Proper-Noun → *Houston* | *NWA*
Aux → *does*
Preposition → *from* | *to* | *on* | *near* | *through*



CNF (binarized) grammar

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Nominal → book | flight | meal | money
Nominal → Nominal Noun
Nominal → Nominal PP
VP → book | include | prefer
VP → Verb NP
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Lexicon
Det → that | this | a
Noun → book | flight | meal | money
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Aux → does
Preposition → from | to | on | near | through

CKY Parsing: Table

- Any constituent can conceivably span [*i*, *j*] for all $0 \leq i < j \leq N$, where *N* = length of input string
  - We need an *N* × *N* table to keep track of all spans…
  - But we only need half of the table
- Semantics of table: cell [*i*, *j*] contains A iff A spans *i* to *j* in the input string
  - Of course, must be allowed by the grammar!

TO:

FROM:	1	2	3	4	5	6
0	0–1	0–2	0–3	0–4	0–5	0–6
1		1–2	1–3	1–4	1–5	1–6
2			2–3	2–4	2–5	2–6
3				3–4	3–5	3–6
4					4–5	4–6
5						5–6

CKY Parsing: Table-Filling

- In order for A to span [*i*, *j*]
  - A $\rightarrow$ B C is a rule in the grammar, and
  - There must be a B in [*i*, *k*] and a C in [*k*, *j*] for some *i*<*k*<*j*

- Operationally
  - To apply rule A $\rightarrow$ B C, look for a B in [*i*, *k*] and a C in [*k*, *j*]
  - In the table: look left in the row and down in the column

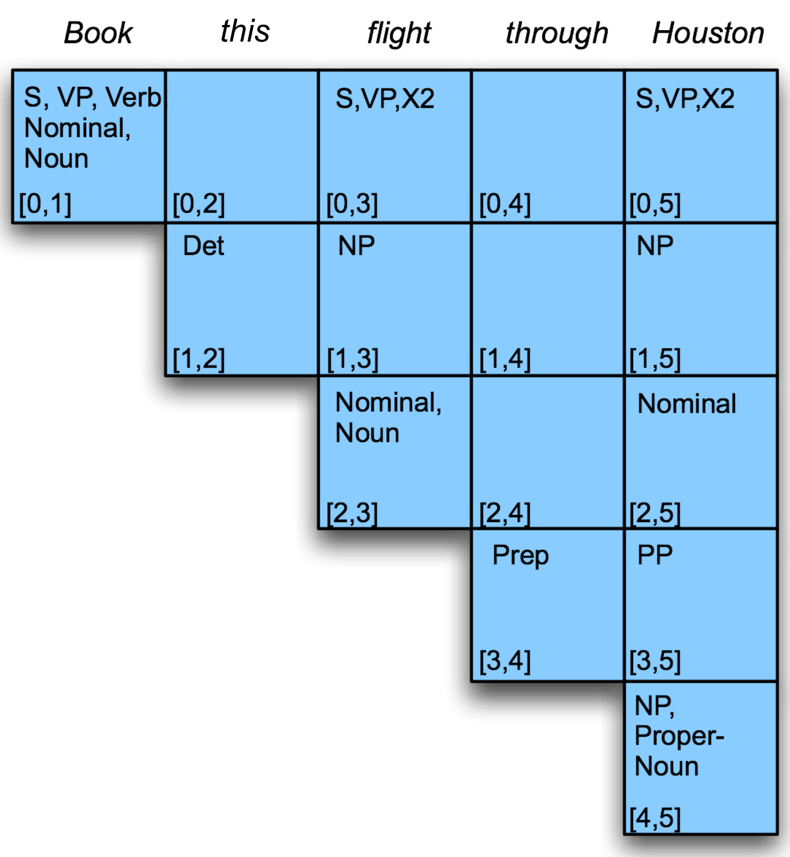
TO:

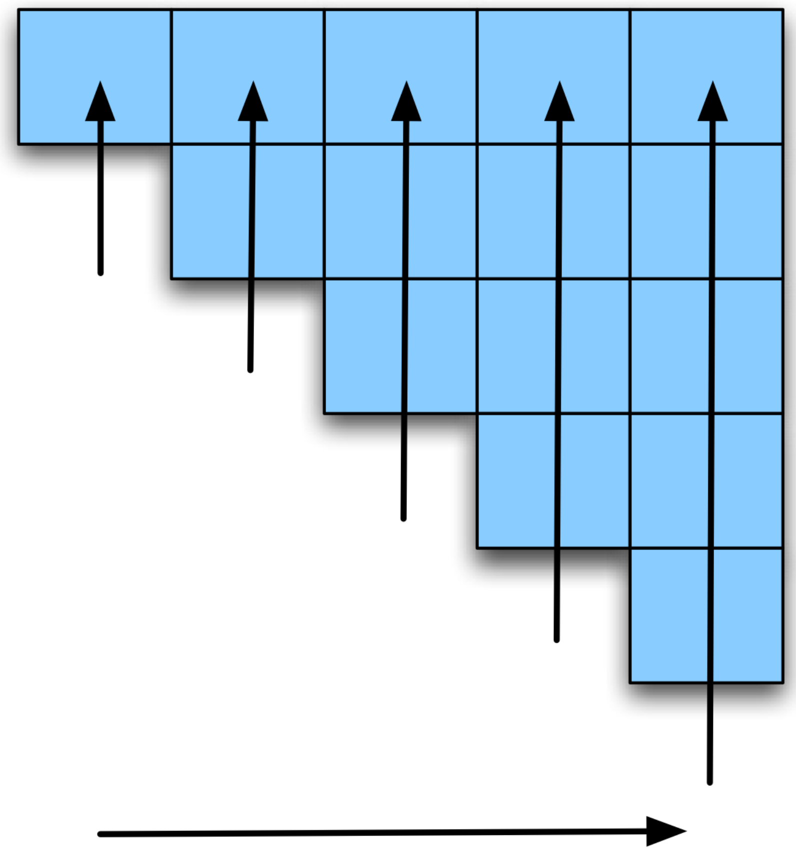
FROM:	1	2	3	4	5	6
0	0–1	0–2	0–3	0–4	0–5	0–6
1		1–2	1–3	1–4	1–5	1–6
2			2–3	2–4	2–5	2–6
3				3–4	3–5	3–6
4					4–5	4–6
5						5–6

CKY Parsing: Canonical Ordering

- Standard CKY algorithm:
  - Fill the table a column at a time, from left to right, bottom to top
  - Whenever we're filling a cell, the parts needed are already in the table (to the left and below)
- Nice property: processes input left to right, word at a time

CKY Parsing: Ordering Illustrated





CKY Algorithm

function CKY-PARSE(*words, grammar*) **returns** *table*

for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**

$table[j-1, j] \leftarrow \{A \mid A \rightarrow words[j] \in grammar\}$

for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**

for $k \leftarrow i+1$ **to** $j-1$ **do**

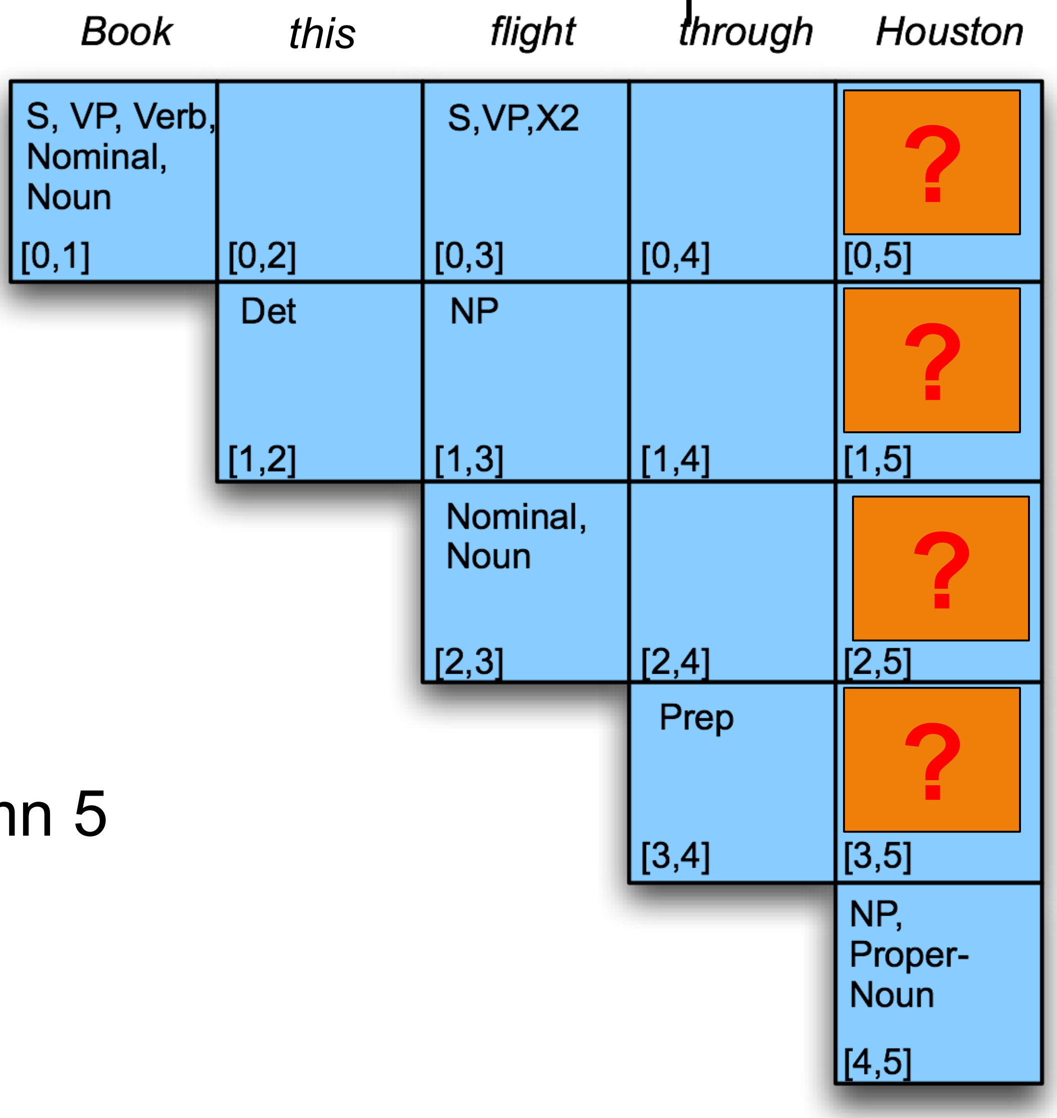
$table[i,j] \leftarrow table[i,j] \cup$

$\{A \mid A \rightarrow BC \in grammar,$

$B \in table[i,k],$

$C \in table[k,j]\}$

CKY: Example



Filling column 5

CKY: Example

Recall our CNF grammar:

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NP → *I* | *she* | *me*
NP → *TWA* | *Houston*
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Nominal → *Nominal Noun*
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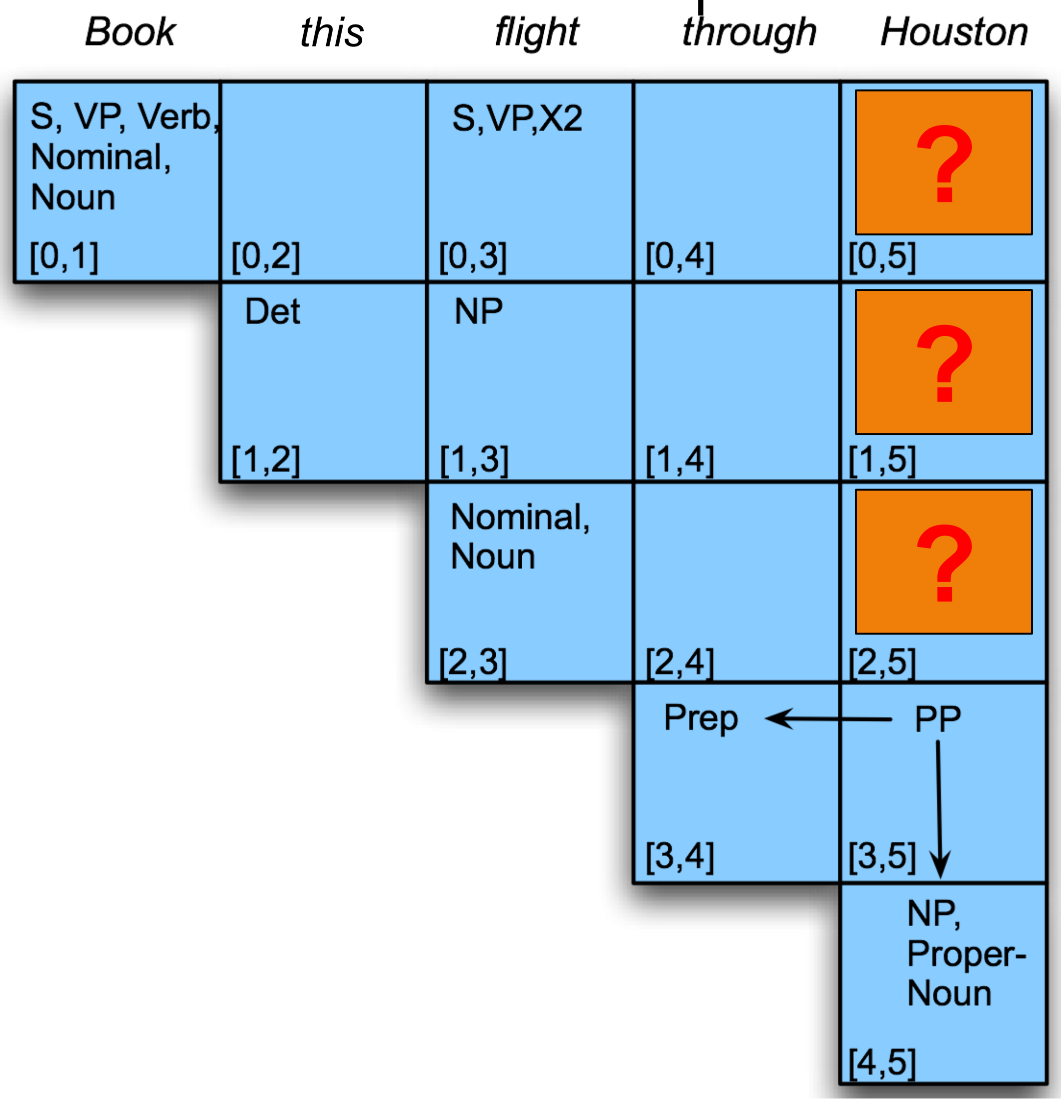
VP → *Verb NP*
VP → *X2 PP*
X2 → *Verb NP*
VP → *Verb PP*
VP → *VP PP*
PP → *Preposition NP*

Book	*this*	*flight*	*through*	*Houston*
S, VP, Verb, Nominal, Noun [0,1]	[0,2]	S,VP,X2 [0,3]	[0,4]	? [0,5]
	Det [1,2]	NP [1,3]	[1,4]	? [1,5]
		Nominal, Noun [2,3]	[2,4]	? [2,5]
			Prep [3,4]	? [3,5]
				NP, Proper-Noun [4,5]

CKY: Example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[1,2]
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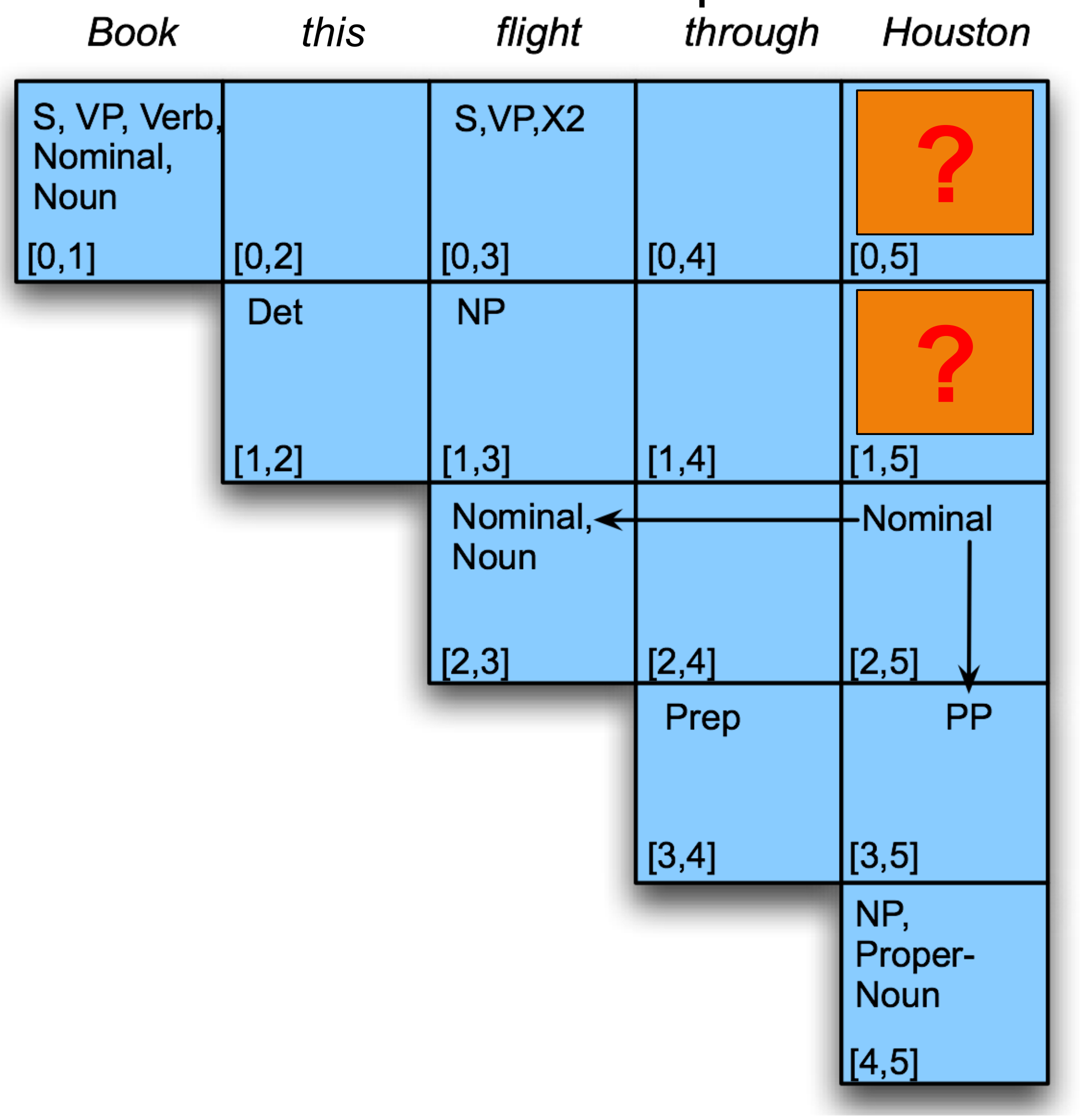
CKY: Example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CKY: Example

Recall our CNF grammar:

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S → book | include | prefer
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S → X2 PP
S → Verb PP
S → VP PP
NP → I | she | me
NP → TWA | Houston
NP → Det Nominal
Nominal → book | flight | meal | money
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X2 → Verb NP
VP → Verb PP
VP → VP PP
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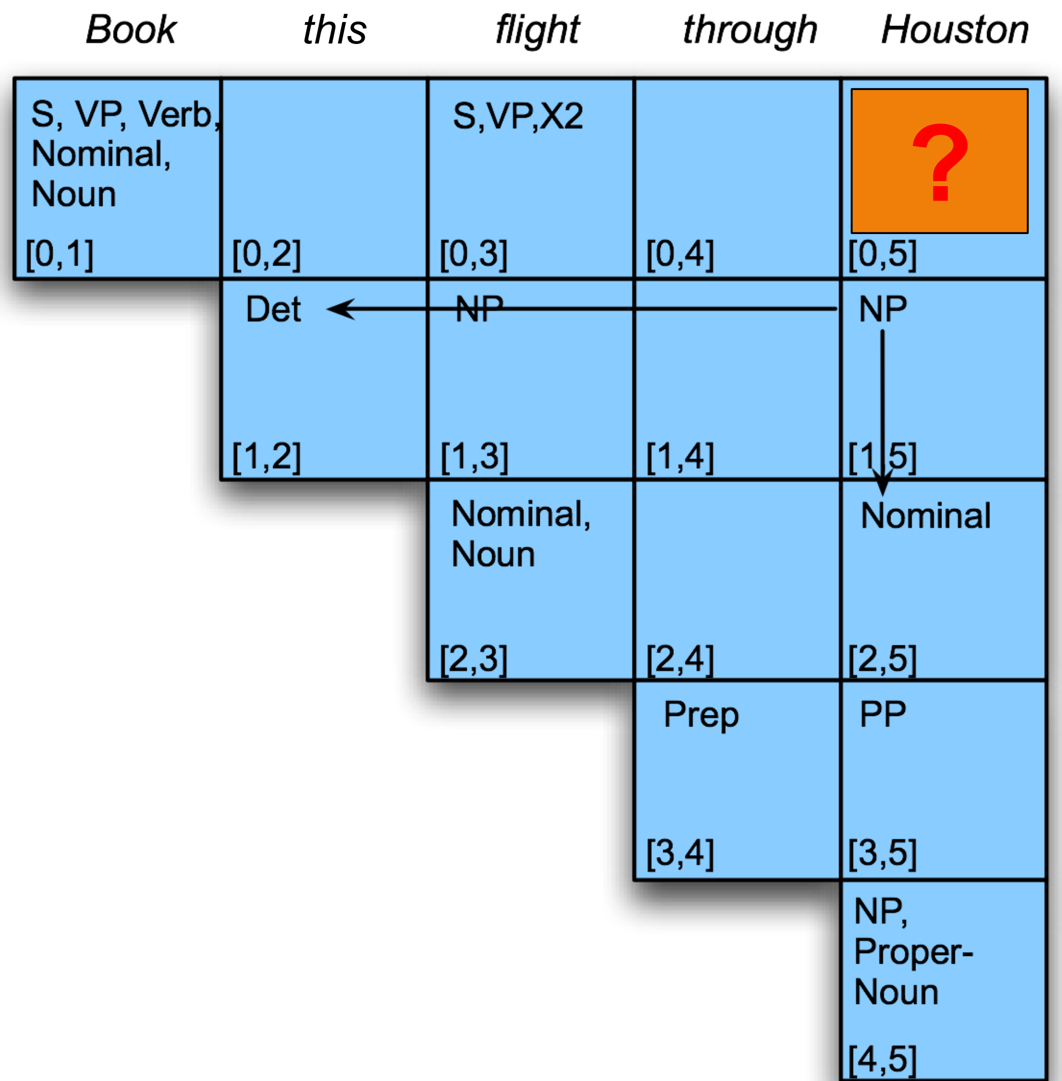
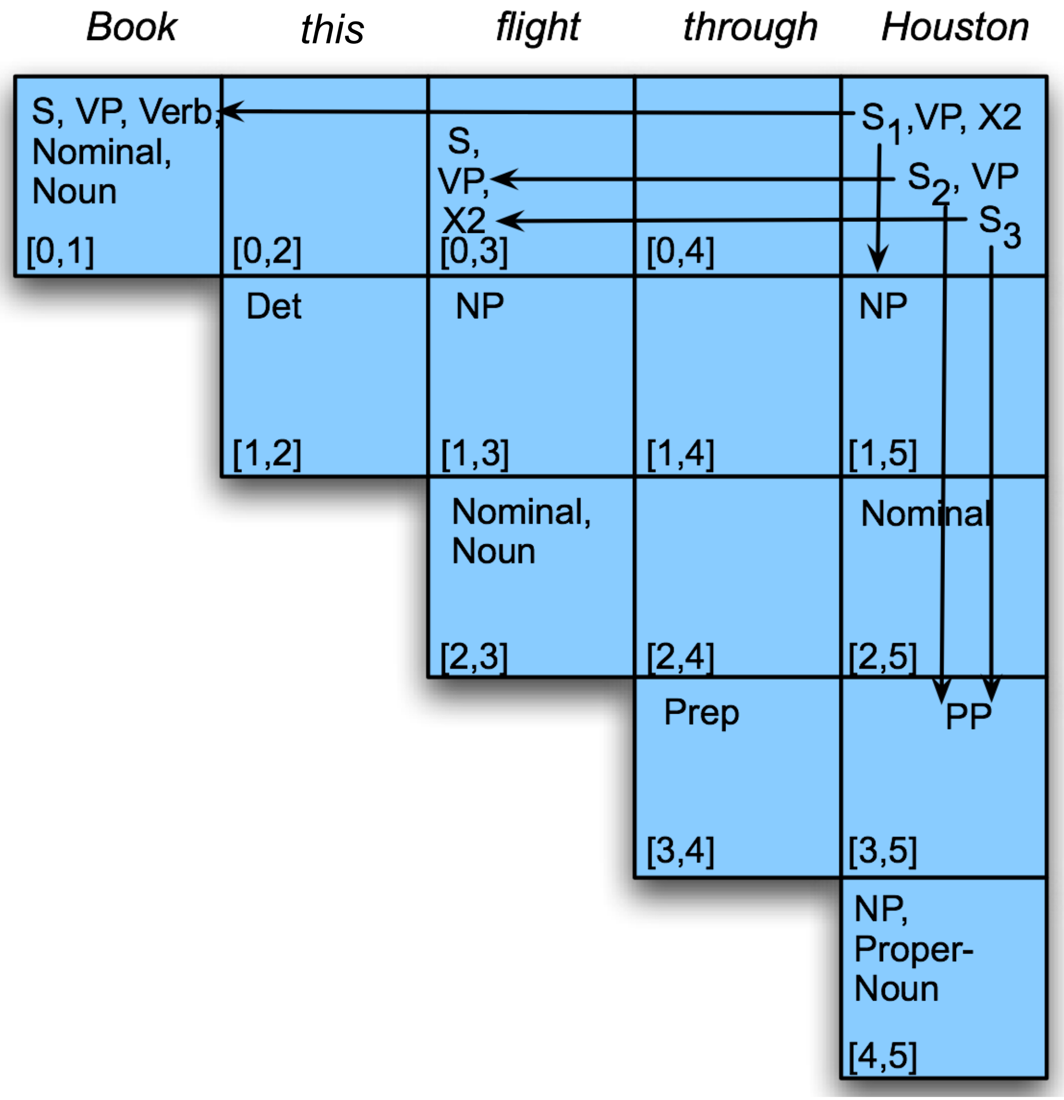


CKY: Example


Book
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S, VP, Verb, Nominal, Noun
[0,1]
[0,2]
S, VP, X2
[0,3]
[0,4]
S1,VP, X2
S2, VP
S3
Det
[1,2]
NP
[1,3]
[1,4]
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CKY Parsing: Recognize or Parse

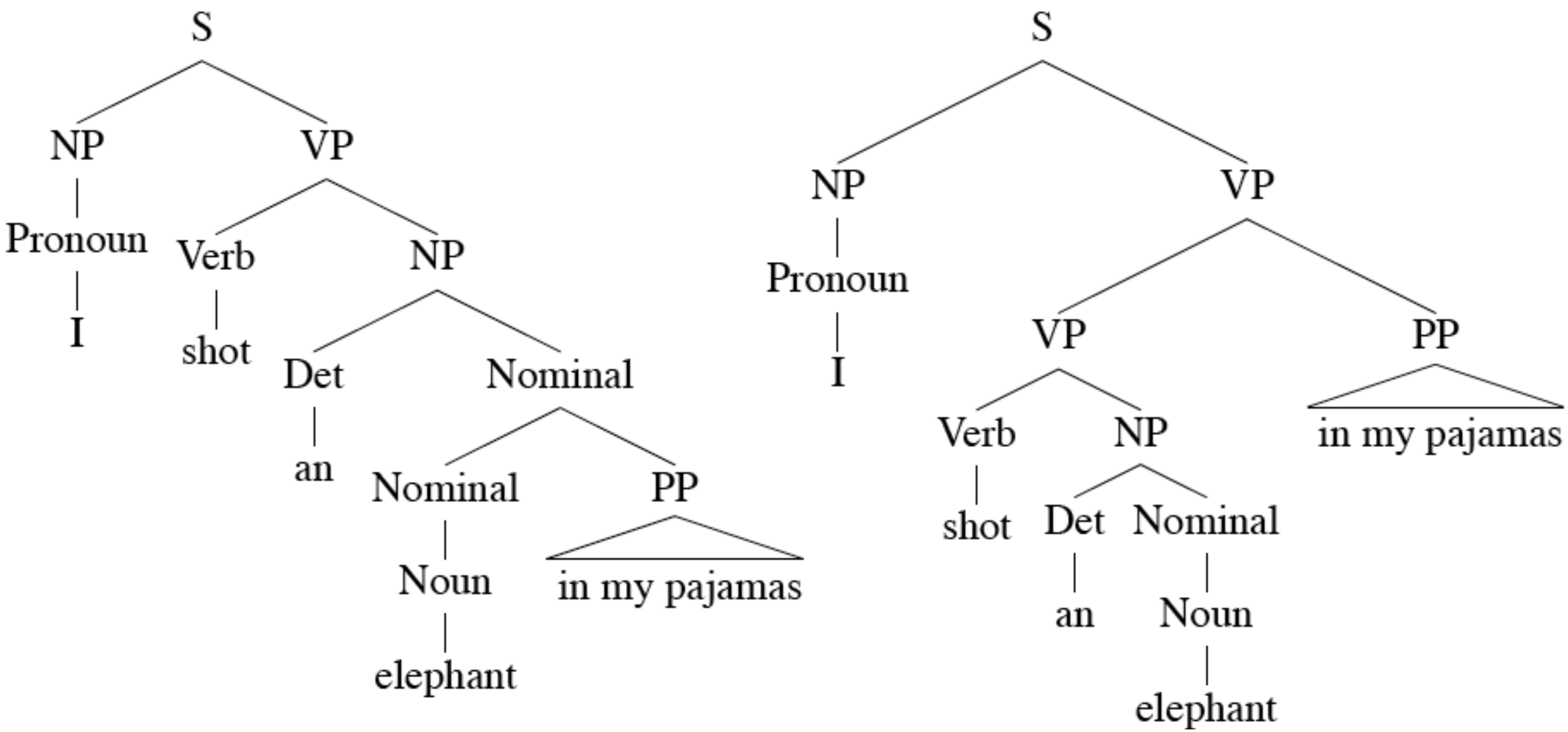
- Recognizer
  - Output is binary
  - Can the complete span of the sentence be covered by an S symbol?

- Parser
  - Output is a parse tree
  - From recognizer to parser: add backpointers!

Ambiguity

- CKY can return multiple parse trees
  - Plus: compact encoding with shared sub-trees
  - Plus: work deriving shared sub-trees is reused
  - Minus: algorithm doesn’t tell us which parse is correct!

Ambiguity


S
NP
VP
Pronoun
Verb
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I
shot
Det
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an
Nominal
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Noun
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PROBABILISTIC CONTEXT-FREE GRAMMARS

Simple Probability Model

- A derivation (tree) consists of the bag of grammar rules that are in the tree
  - The probability of a tree is the product of the probabilities of the rules in the derivation.

$$P(T,S) = \prod_{node \in T} P(rule(n))$$

Rule Probabilities

- What's the probability of a rule?
- Start at the top...
  - A tree should have an *S* at the top. So given that we know we need an *S*, we can ask about the probability of each particular *S* rule in the grammar: P(particular rule | S)
- In general we need $P(\alpha \rightarrow \beta \mid \alpha)$ for each rule in the grammar

Training the Model

- We can get the estimates we need from a treebank

$$P(\alpha \rightarrow \beta|\alpha) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{Count}(\alpha \rightarrow \gamma)} = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$$

For example, to get the probability for a particular *VP* rule:

1. count all the times the rule is used
2. divide by the number of *VP*s overall.

Parsing (Decoding)

How can we get the best (most probable) parse for a given input?

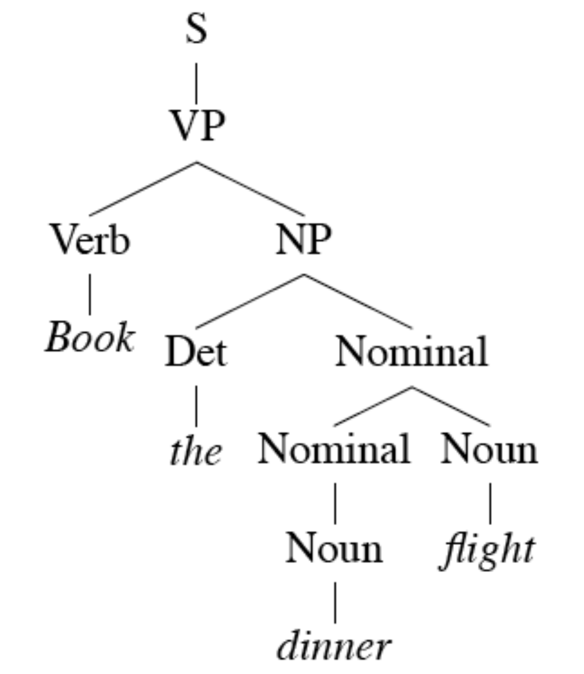
1. Enumerate all the trees for a sentence

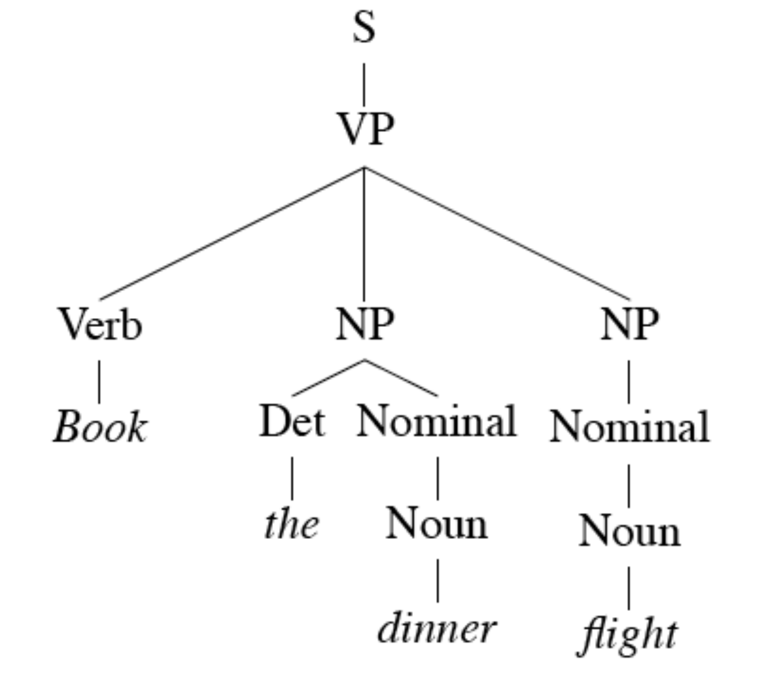
2. Assign a probability to each using the model

3. Return the argmax

Example

- Consider…
  - *Book the dinner flight*





Examples

- These trees consist of the following rules.

Rules			P	Rules			P
S	→	VP	.05	S	→	VP	.05
VP	→	Verb NP	.20	VP	→	Verb NP NP	.10
NP	→	Det Nominal	.20	NP	→	Det Nominal	.20
Nominal	→	Nominal Noun	.20	NP	→	Nominal	.15
Nominal	→	Noun	.75	Nominal	→	Noun	.75
				Nominal	→	Noun	.75
Verb	→	book	.30	Verb	→	book	.30
Det	→	the	.60	Det	→	the	.60
Noun	→	dinner	.10	Noun	→	dinner	.10
Noun	→	flights	.40	Noun	→	flights	.40

$$P(T_{left}) = .05 * .20 * .20 * .20 * .75 * .30 * .60 * .10 * .40 = \mathbf{2.2 \times 10^{-6}}$$
$$P(T_{right}) = .05 * .10 * .20 * .15 * .75 * .75 * .30 * .60 * .10 * .40 = \mathbf{6.1 \times 10^{-7}}$$

Dynamic Programming

- Of course, as with normal parsing we don’t really want to do it that way…
- Instead, we need to exploit dynamic programming
  - For the parsing (as with CKY)
  - And for computing the probabilities and returning the best parse (as with Viterbi and HMMs)

Probabilistic CKY

- Store probabilities of constituents in the table
  - table[i,j,A] = probability of constituent A that spans positions i through j in input

- If *A* is derived from the rule $A \rightarrow B\ C$:
  - table[i,j,A] = P($A \rightarrow B\ C \mid A$) * table[i,k,*B*] * table[k,j,*C*]
  - Where
    - P($A \rightarrow B\ C \mid A$) is the rule probability
    - table[i,k,*B*] and table[k,j,*C*] are already in the table given the way that CKY operates

- Only store the MAX probability over all the A rules.

Probabilistic CKY

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
                                                   and its probability
  for j ← from 1 to LENGTH(words) do
      for all { A | A → words[j] ∈ grammar}
         table[j−1, j, A] ← P(A → words[j])
      for i ← from j−2 downto 0 do
          for k ← i+1 to j−1 do
                for all { A | A → BC ∈ grammar,
                                and table[i,k,B] > 0 and table[k, j,C] > 0 }
                      if (table[i,j,A] < P(A → BC) × table[i,k,B] × table[k,j,C]) then
                          table[i,j,A] ← P(A → BC) × table[i,k,B] × table[k,j,C]
                          back[i,j,A] ← {k,B,C}
      return BUILD_TREE(back[1, LENGTH(words), S]), table[1, LENGTH(words), S]
```

Problems with PCFGs

- The probability model we're using is just based on the bag of rules in the derivation…
  1. Doesn't take the actual words into account in any useful way.
  2. Doesn't take into account *where* in the derivation a rule is used
  3. *Doesn't work terribly well*

IMPROVING OUR PARSER

Improved Approaches

There are two approaches to overcoming these shortcomings

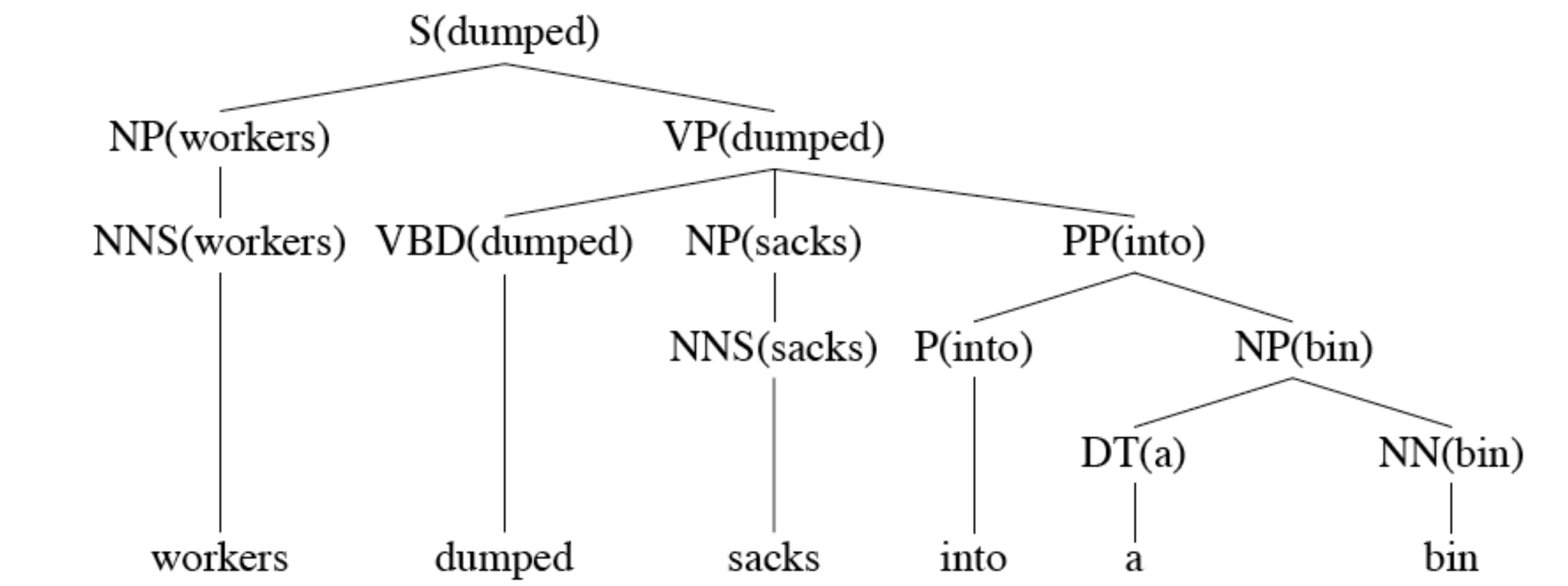
1. Rewrite the grammar to better capture the dependencies among rules

2. Integrate lexical dependencies into the model

Solution 2:
Lexicalized Grammars

- Lexicalize the grammars with heads
- Compute the rule probabilities on these lexicalized rules
- Run Prob CKY as before

Lexicalized Grammars: Example



How can we learn probabilities for lexicalized rules?

- We used to have
  - VP -> V NP PP
  - P(rule|VP) = count of this rule divided by the number of VPs in a treebank

- Now we have fully lexicalized rules…
  - VP(dumped)-> V(dumped) NP(sacks)PP(into)

P(r|VP ^ dumped is the verb ^ sacks is the head of the NP ^ into is the head of the PP)

We need to make independence assumptions

- Strategies: exploit independence and collect the statistics we can get
  - Many many ways to do this...

- Let’s consider one generative story: given a rule we’ll

1. Generate the head
2. Generate the stuff to the left of the head
3. Generate the stuff to the right of the head

From the generative story to rule probabilities…

The rule probability for

$$P(VP(dumped,VBD) \rightarrow VBD(dumped,\ VBD)\ \ NP(sacks,NNS)\ \ PP(into,P))$$

Can be estimated as

$$\begin{aligned} P_H(VBD|VP,\ dumped) \ &\times\ P_L(STOP|VP,VBD,dumped) \\ &\times\ P_R(NP(sacks,NNS)|VP,VBD,dumped) \\ &\times\ P_R(PP(into,P)|VP,VBD,dumped) \\ &\times\ P_R(STOP|VP,VBD,dumped) \end{aligned}$$

Framework

- That's just one simple model
  - "Collins Model 1"
- Other assumptions that might lead to better models
  - make sure that you can get the counts you need
  - make sure they can get exploited efficiently during decoding

Today's Agenda

- Grammar-based parsing with CFGs
  - CKY algorithm
- Dealing with ambiguity
  - Probabilistic CFGs
- Strategies for improvement
  - Lexicalization

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- Tools for parsing English, Chinese, French, … with PCFGs http://nlp.stanford.edu/software/lex-parser.shtml