Empirical Methods in Natural Language Processing
Lecture 5
N-gram Language Models

(most slides from Sharon Goldwater; some adapted from Alex Lascarides)

29 January 2017

Recap

- Previously, we talked about corpus data and some of the information we can get from it, like word frequencies.
- For some tasks, like sentiment analysis, word frequencies alone can work pretty well (though can certainly be improved on).
- For other tasks, we need more.
- Today we consider **sentence probabilities**: what are they, why are they useful, and how might we compute them?

Review: Word-based sentiment

- Recall that we can predict sentiment for a document based on counting positive and negative words.

- Do you think the following words would be positive or negative in a movie review?
  - OK
  - Action
  - Star

N-grams

- In some cases, looking at more than one word at a time might be more informative.
  - action movie vs. action packed
  - Star Wars vs. star studded

- An n-**gram** is a word sequence of length n.
  - 1-gram or **unigram**: action
  - 2-gram or **bigram**: action packed
  - 3-gram or **trigram**: action packed adventure
  - 4-gram: action packed adventure film

N-grams

The Force Awakens brings back the Old Trilogy 's heart , humor , mystery , and fun .

How many:

- Unigrams?
- Bigrams?
- Trigrams?

Character N-grams

- A **character n-gram** applies the same idea to characters rather than words.
- E.g. unnatural has character bigrams un, nn, na, . . . , al
- Why might this concept be useful for NLP?

Towards Sentence Probabilities

- "Probability of a sentence" = how likely is it to occur in natural language
  - Consider only a specific language (English)

$$\text{P(the cat slept peacefully)} > \text{P(slept the peacefully cat)}$$

$$\text{P(she studies morphosyntax)} > \text{P(she studies more faux syntax)}$$

Language models in NLP

- It's very difficult to know the true probability of an arbitrary sequence of words.
- But we can define a **language model** that will give us good approximations.
- Like all models, language models will be good at capturing some things and less good for others.
  - We might want different models for different tasks.
  - Today, one type of language model: an **N-gram model**.

Spelling correction

Sentence probabilities help decide correct spelling.

mis-spelled text		no much effert
↓	(Error model)	
possible outputs		no much effect so much effort no much effort not much effort ...
↓	(Language model)	
best-guess output		not much effort

Automatic speech recognition

Sentence probabilities help decide between similar-sounding options.

speech input

↓ (Acoustic model)

possible outputs

- She studies morphosyntax
- She studies more faux syntax
- She's studies morph or syntax
- ...

↓ (Language model)

best-guess output: She studies morphosyntax

Machine translation

Sentence probabilities help decide word choice and word order.

non-English input		
↓	(Translation model)	
possible outputs		She is going home She is going house She is traveling to home To home she is going ...
↓	(Language model)	
best-guess output		She is going home

LMs for prediction

- LMs can be used for **prediction** as well as correction.

- Ex: predictive text correction/completion on your mobile phone.
  - Keyboard is tiny, easy to touch a spot slightly off from the letter you meant.
  - Want to correct such errors as you go, and also provide possible completions. Predict as as you are typing: ineff...

- In this case, LM may be defined over sequences of *characters* instead of (or in addition to) sequences of words.

But how to estimate these probabilities?

- We want to know the probability of word sequence $\vec{w} = w_1 \ldots w_n$ occurring in English.

- Assume we have some **training data**: large corpus of general English text.

- We can use this data to **estimate** the probability of $\vec{w}$ (even if we never see it in the corpus!)

Probability theory vs estimation

- Probability theory can solve problems like:
  - I have a jar with 6 blue marbles and 4 red ones.
  - If I choose a marble uniformly at random, what's the probability it's red?

Probability theory vs estimation

- Probability theory can solve problems like:
  - I have a jar with 6 blue marbles and 4 red ones.
  - If I choose a marble uniformly at random, what's the probability it's red?
- But often we don't know the true probabilities, only have data:
  - I have a jar of marbles.
  - I repeatedly choose a marble uniformly at random and then replace it before choosing again.
  - In ten draws, I get 6 blue marbles and 4 red ones.
  - On the next draw, what's the probability I get a red marble?
- The latter also requires estimation theory.

Notation

- I will often omit the random variable in writing probabilities, using $P(x)$ to mean $P(X = x)$.

- When the distinction is important, I will use
  - $P(x)$ for *true* probabilities
  - $\hat{P}(x)$ for *estimated* probabilities
  - $P_{\mathrm{E}}(x)$ for estimated probabilities using a particular estimation method E.

- But since we almost always mean estimated probabilities, may get lazy later and use $P(x)$ for those too.

Example estimation: M&M colors

What is the proportion of each color of M&M?

- In 48 packages, I find[1] 2620 M&Ms, as follows:

Red	Orange	Yellow	Green	Blue	Brown
372	544	369	483	481	371

- How to estimate probability of each color from this data?

[1] Actually, data from: https://joshmadison.com/2007/12/02/mms-color-distribution-analysis/

Relative frequency estimation

- Intuitive way to estimate discrete probabilities:

$$P_{\mathrm{RF}}(x) = \frac{C(x)}{N}$$

where $C(x)$ is the count of x in a large dataset, and $N = \sum_{x'} C(x')$ is the total number of items in the dataset.

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- M&M example: $P_{\mathrm{RF}}(\text{red}) = \frac{372}{2620} = .142$

- This method is also known as **maximum-likelihood estimation** (MLE) for reasons we'll get back to.

MLE for sentences?

Can we use MLE to estimate the probability of $\vec{w}$ as a sentence of English? That is, the prob that some sentence S has words $\vec{w}$?

$$P_{\text{MLE}}(S = \vec{w}) = \frac{C(\vec{w})}{N}$$

where $C(\vec{w})$ is the count of $\vec{w}$ in a large dataset, and
N is the total number of sentences in the dataset.

Sentences that have never occurred

the Archaeopteryx soared jaggedly amidst foliage

vs

jaggedly trees the on flew

- Neither ever occurred in a corpus (until I wrote these slides).
  ⇒ $C(\vec{w}) = 0$ in both cases: MLE assigns both zero probability.

- But one is grammatical (and meaningful), the other not.
  ⇒ Using MLE on full sentences doesn't work well for language model estimation.

The problem with MLE

- MLE thinks anything that hasn't occurred will never occur (P=0).
- Clearly not true! Such things can have differering, and non-zero, probabilities:
  - My hair turns blue
  - I injure myself in a skiing accident
  - I travel to Finland
- And similarly for word sequences that have never occurred.

Sparse data

- In fact, even things that occur once or twice in our training data are a problem. Remember these words from Europarl?

  cornflakes, mathematicians, pseudo-rapporteur, lobby-ridden, Lycketoft, UNCITRAL, policyfor, Commissioneris, 145.95

  All occurred once. Is it safe to assume all have equal probability?

- This is a **sparse data** problem: not enough observations to estimate probabilities well. (Unlike the M&Ms, where we had large counts for all colours!)

- For sentences, many (most!) will occur rarely if ever in our training data. So we need to do something smarter.

Towards better LM probabilities

- One way to try to fix the problem: estimate $P(\vec{w})$ by combining the probabilities of smaller parts of the sentence, which will occur more frequently.
- This is the intuition behind **N-gram language models**.

Deriving an N-gram model

- We want to estimate $P(S = w_1 \ldots w_n)$.
  - Ex: $P(S = \text{the cat slept quietly})$.

- This is really a joint probability over the words in S:
  $P(W_1 = \text{the}, W_2 = \text{cat}, W_3 = \text{slept}, \ldots W_4 = \text{quietly})$.

- Concisely, $P(\text{the, cat, slept, quietly})$ or $P(w_1, \ldots w_n)$.

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- Recall that for a joint probability, $P(X, Y) = P(Y|X)P(X)$. So,

  $P(\text{the, cat, slept, quietly}) = P(\text{quietly}|\text{the, cat, slept})P(\text{the, cat, slept})$
  $= P(\text{quietly}|\text{the, cat, slept})P(\text{slept}|\text{the, cat})P(\text{the, cat})$
  $= P(\text{quietly}|\text{the, cat, slept})P(\text{slept}|\text{the, cat})P(\text{cat}|\text{the})P(\text{the})$

Deriving an N-gram model

- More generally, the chain rule gives us:

$$P(w_1, \ldots w_n) = \prod_{i=1}^{n} P(w_i | w_1, w_2, \ldots w_{i-1})$$

- But many of these conditional probs are just as sparse!
  - If we want $P($I spent three years before the mast$)$...
  - we still need $P($mast$|$I spent three years before the$)$.

Example due to Alex Lascarides/Henry Thompson

Deriving an N-gram model

- So we make an **independence assumption**: the probability of a word only depends on a fixed number of previous words (**history**).
  - **trigram model**: $P(w_i|w_1, w_2, \ldots w_{i-1}) \approx P(w_i|w_{i-2}, w_{i-1})$
  - **bigram model**: $P(w_i|w_1, w_2, \ldots w_{i-1}) \approx P(w_i|w_{i-1})$
  - **unigram model**: $P(w_i|w_1, w_2, \ldots w_{i-1}) \approx P(w_i)$

- In our example, a trigram model says
  - $P(\text{mast}|\text{I spent three years before the}) \approx P(\text{mast}|\text{before the})$

Trigram independence assumption

- Put another way, trigram model assumes these are all equal:
  - $P(\text{mast}|\text{I spent three years before the})$
  - $P(\text{mast}|\text{I went home before the})$
  - $P(\text{mast}|\text{I saw the sail before the})$
  - $P(\text{mast}|\text{I revised all week before the})$

  because all are estimated as $P(\text{mast}|\text{before the})$

- Also called a **Markov assumption**

  Andrey Markov $\rightarrow$ 

- Not always a good assumption! But it does reduce the sparse data problem.

Estimating trigram conditional probs

- We still need to estimate $P(\text{mast}|\text{before, the})$: the probability of mast given the two-word history before, the.

- If we use relative frequencies (MLE), we consider:
  - Out of all cases where we saw before, the as the first two words of a trigram,
  - how many had mast as the third word?

Estimating trigram conditional probs

- So, in our example, we'd estimate

$$P_{MLE}(\text{mast}|\text{before, the}) = \frac{C(\text{before, the, mast})}{C(\text{before, the})}$$

where $C(x)$ is the count of x in our training data.

- More generally, for any trigram we have

$$P_{MLE}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}$$

Example from *Moby Dick* corpus

$$C(\textit{before, the}) = 55$$

$$C(\textit{before, the, mast}) = 4$$

$$\frac{C(\textit{before, the, mast})}{C(\textit{before, the})} = 0.0727$$

- *mast* is the second most common word to come after *before the* in *Moby Dick*; *wind* is the most frequent word.
- $P_{MLE}(\textit{mast})$ is 0.00049, and $P_{MLE}(\textit{mast}|\textit{the})$ is 0.0029.
- So seeing *before the* vastly increases the probability of seeing *mast* next.

Trigram model: summary

- To estimate $P(\vec{w})$, use chain rule and make an indep. assumption:

$$P(w_1, \ldots w_n) = \prod_{i=1}^{n} P(w_i | w_1, w_2, \ldots w_{i-1})$$

$$\approx P(w_1) P(w_2 | w_1) \prod_{i=3}^{n} P(w_i | w_{i-2}, w_{w-1})$$

- Then estimate each trigram prob from data (here, using MLE):

$$P_{MLE}(w_i | w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}$$

- On your own: work out the equations for other N-grams (e.g., bigram, unigram).

Practical details (1)

- Trigram model assumes two word history:

$$P(\vec{w}) = P(w_1)P(w_2|w_1)\prod_{i=3}^{n} P(w_i|w_{i-2}, w_{w-1})$$

- But consider these sentences:

	w_1	w_2	w_3	w_4
(1)	he	saw	the	yellow
(2)	feeds	the	cats	daily

- What's wrong? Does the model capture these problems?

Beginning/end of sequence

- To capture behaviour at beginning/end of sequences, we can augment the input:

	w_{-1}	w_0	w_1	w_2	w_3	w_4	w_5
(1)	<s>	<s>	he	saw	the	yellow	</s>
(2)	<s>	<s>	feeds	the	cats	daily	</s>

- That is, assume $w_{-1} = w_0 =$ <s> and $w_{n+1} =$ </s> so:

$$P(\vec{w}) = \prod_{i=1}^{n+1} P(w_i | w_{i-2}, w_{i-1})$$

- Now, $P(\texttt{</s>}|\texttt{the, yellow})$ is low, indicating this is not a good sentence.

Beginning/end of sequence

- Alternatively, we could model all sentences as one (very long) sequence, including punctuation:

  two cats live in sam 's barn . sam feeds the cats daily . yesterday , he saw the yellow cat catch a mouse . [...]

- Now, trigrams like $P(.|\texttt{cats daily})$ and $P(,|\texttt{. yesterday})$ tell us about behavior at sentence edges.

- Here, all tokens are lowercased. What are the pros/cons of *not* doing that?

Practical details (2)

- Word probabilities are typically very small.

- Multiplying lots of small probabilities quickly gets so tiny we can't represent the numbers accurately, even with double precision floating point.

- So in practice, we typically use **negative log probabilities** (sometimes called **costs**):
  - Since probabilities range from 0 to 1, negative log probs range from 0 to ∞: *lower* cost = *higher* probability.
  - Instead of *multiplying* probabilities, we *add* neg log probabilities.

Interim Summary: N-gram probabilities

- “Probability of a sentence”: how likely is it to occur in natural language? Useful in many natural language applications.

- We can never know the true probability, but we may be able to estimate it from corpus data.

- N-gram models are one way to do this:
  - To alleviate sparse data, assume word probs depend only on short history.
  - Tradeoff: longer histories may capture more, but are also more sparse.
  - So far, we estimated N-gram probabilites using MLE.

Interim Summary: Language models

- **Language models** tell us $P(\vec{w}) = P(w_1 \ldots w_n)$: *How likely to occur is this sequence of words?*

  Roughly: *Is this sequence of words a "good" one in my language?*

- LMs are used as a component in applications such as speech recognition, machine translation, and predictive text completion.

- To reduce sparse data, N-gram LMs assume words depend only on a fixed-length history, even though we know this isn't true.

Coming up next:

- Weaknesses of MLE and ways to address them (more issues with sparse data).

- How to evaluate a language model: are we estimating sentence probabilities accurately?

Evaluating a language model

- Intuitively, a trigram model captures more context than a bigram model, so should be a “better” model.
- That is, it should more accurately predict the probabilities of sentences.
- But how can we measure this?

Two types of evaluation in NLP

- **Extrinsic**: measure performance on a downstream application.
  - For LM, plug it into a machine translation/ASR/etc system.
  - The most reliable evaluation, but can be time-consuming.
  - And of course, we still need an evaluation measure for the downstream system!

- **Intrinsic**: design a measure that is inherent to the current task.
  - Can be much quicker/easier during development cycle.
  - But not always easy to figure out what the right measure is: ideally, one that correlates well with extrinsic measures.

Let's consider how to define an intrinsic measure for LMs.

Entropy

- Definition of the **entropy** of a random variable X:

  $H(X) = \sum_x -P(x)\ \log_2 P(x)$

- Intuitively: a measure of uncertainty/disorder

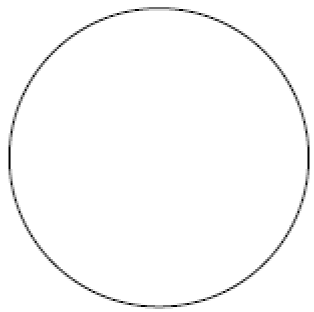
- Also: the expected value of $-\log_2 P(X)$

Entropy Example

One event (outcome)

$P(a) = 1$

$$H(X) = -1 \log_2 1$$
$$= 0$$

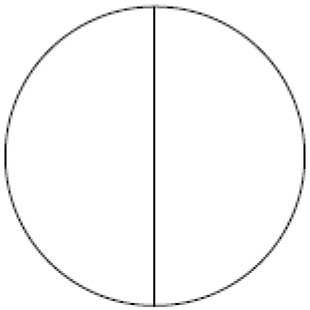


Entropy Example

2 equally likely events:

$$P(a) = 0.5$$
$$P(b) = 0.5$$

$$\begin{aligned} H(X) &= -0.5 \log_2 0.5 - 0.5 \log_2 0.5 \\ &= -\log_2 0.5 \\ &= 1 \end{aligned}$$

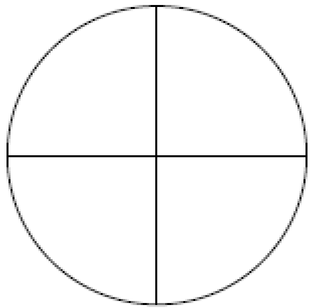


Entropy Example

4 equally likely events:

$$P(a) = 0.25$$
$$P(b) = 0.25$$
$$P(c) = 0.25$$
$$P(d) = 0.25$$

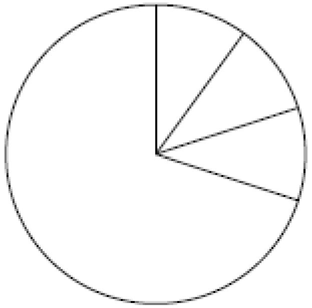
$$\begin{aligned} H(X) = & -0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\ & -0.25 \log_2 0.25 - 0.25 \log_2 0.25 \\ = & -\log_2 0.25 \\ = & \; 2 \end{aligned}$$



Entropy Example

3 equally likely events and one more likely than the others:

$P(a) = 0.7$
$P(b) = 0.1$
$P(c) = 0.1$
$P(d) = 0.1$

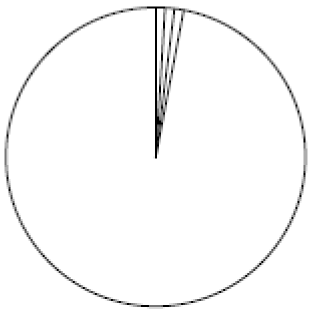


$$
\begin{aligned}
H(X) = & - 0.7 \log_2 0.7 - 0.1 \log_2 0.1 \\
& - 0.1 \log_2 0.1 - 0.1 \log_2 0.1 \\
= & - 0.7 \log_2 0.7 - 0.3 \log_2 0.1 \\
= & - (0.7)(-0.5146) - (0.3)(-3.3219) \\
= & \; 0.36020 + 0.99658 \\
= & \; 1.35678
\end{aligned}
$$

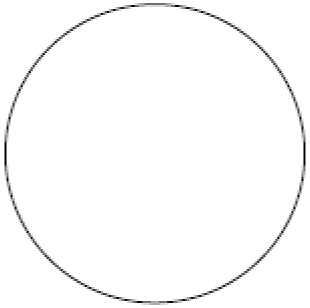
Entropy Example

3 equally likely events and one much more likely than the others:

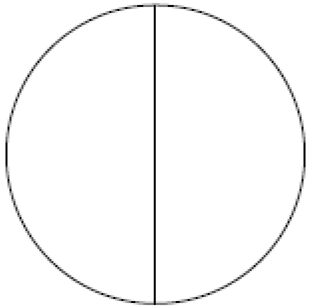
$$P(a) = 0.97$$
$$P(b) = 0.01$$
$$P(c) = 0.01$$
$$P(d) = 0.01$$



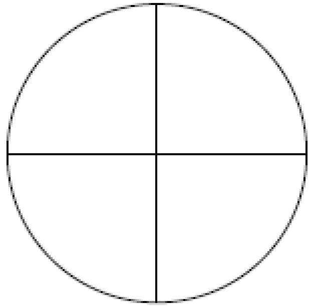
$$\begin{aligned} H(X) = & -0.97 \log_2 0.97 - 0.01 \log_2 0.01 \\ & -0.01 \log_2 0.01 - 0.01 \log_2 0.01 \\ = & -0.97 \log_2 0.97 - 0.03 \log_2 0.01 \\ = & -(0.97)(-0.04394) - (0.03)(-6.6439) \\ = & \, 0.04262 + 0.19932 \\ = & \, 0.24194 \end{aligned}$$



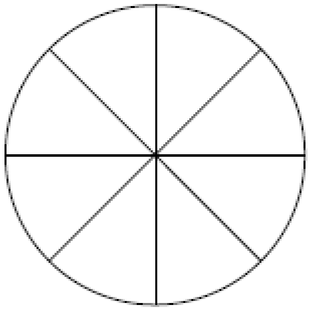
$H(X) = 0$



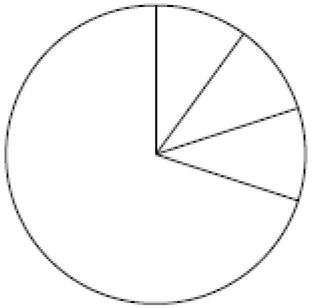
$H(X) = 1$



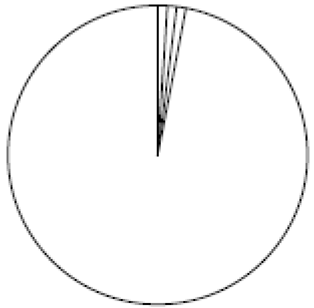
$H(X) = 2$



$H(X) = 3$



$H(X) = 1.35678$



$H(X) = 0.24194$

Entropy as y/n questions

How many yes-no questions (bits) do we need to find out the outcome?

- Uniform distribution with 2^n outcomes: n q's.
- Other cases: entropy is the average number of questions per outcome in a (very) long sequence of outcomes, where questions can consider multiple outcomes at once.

Entropy as encoding sequences

- Assume that we want to encode a sequence of events X.
- Each event is encoded by a sequence of bits, we want to use as few bits as possible.
- For example
  - Coin flip: heads = 0, tails = 1
  - 4 equally likely events: a = 00, b = 01, c = 10, d = 11
  - 3 events, one more likely than others: a = 0, b = 10, c = 11
  - Morse code: *e* has shorter code than *q*
- Average number of bits needed to encode $X \geq$ entropy of X

The Entropy of English

- Given the start of a text, can we guess the next word?
- For humans, the measured entropy is only about 1.3.
  - Meaning: on average, given the preceding context, a human would need only 1.3 y/n questions to determine the next word.
  - This is an upper bound on the true entropy, which we can never know (because we don't know the true probability distribution).
- But what about N-gram models?

Cross-entropy

- Our LM estimates the probability of word sequences.
- A good model assigns high probability to sequences that actually have high probability (and low probability to others).
- Put another way, our model should have low uncertainty (entropy) about which word comes next.
- We can measure this using **cross-entropy**.
- Note that cross-entropy $\geq$ entropy: our model's uncertainty can be no less than the true uncertainty.

Computing cross-entropy

- For $w_1 \dots w_n$ with large n, per-word cross-entropy is well approximated by:

$$H_M(w_1 \dots w_n) = -\frac{1}{n} \log_2 P_M(w_1 \dots w_n)$$

- This is just the average negative log prob our model assigns to each word in the sequence. (i.e., normalized for sequence length).

- Lower cross-entropy $\Rightarrow$ model is better at predicting next word.

Cross-entropy example

Using a bigram model from Moby Dick, compute per-word cross-entropy of I spent three years before the mast (here, without using end-of sentence padding):

$$
\begin{aligned}
& -\tfrac{1}{7}(&& \lg_2(P(\textit{I})) + \lg_2(P(\textit{spent}|\textit{I})) + lg_2(P(\textit{three}|\textit{spent})) + \lg_2(P(\textit{years}|\textit{three})) \\
& && + \lg_2(P(\textit{before}|\textit{years})) + \lg_2(P(\textit{the}|\textit{before})) + \lg_2(P(\textit{mast}|\textit{the})) \quad) \\
= \; & -\tfrac{1}{7}(&& -6.9381 - 11.0546 - 3.1699 - 4.2362 - 5.0 - 2.4426 - 8.4246 \quad) \\
= \; & -\tfrac{1}{7}(&& 41.2660 \quad) \\
\approx \; & \quad 6
\end{aligned}
$$

- Per-word cross-entropy of the *unigram* model is about 11.

- So, unigram model has about 5 bits more uncertainty per word then bigram model. But, what does that mean?

Data compression

- If we designed an optimal code based on our bigram model, we could encode the entire sentence in about 42 bits.

- A code based on our unigram model would require about 77 bits.

- ASCII uses an average of 24 bits per word (168 bits total)!

- So better language models can also give us better data compression: as elaborated by the field of **information theory**.

Perplexity

- LM performance is often reported as **perplexity** rather than cross-entropy.
- Perplexity is simply $2^{\text{cross-entropy}}$
- The average branching factor at each decision point, if our distribution were uniform.
- So, 6 bits cross-entropy means our model perplexity is $2^6 = 64$: equivalent uncertainty to a uniform distribution over 64 outcomes.

Interpreting these measures

I measure the cross-entropy of my LM on some corpus as 5.2.
Is that good?

Interpreting these measures

I measure the cross-entropy of my LM on some corpus as 5.2.
Is that good?

- No way to tell! Cross-entropy depends on both the model and the corpus.
  - Some language is simply more predictable (e.g. casual speech vs academic writing).
  - So lower cross-entropy could mean the corpus is "easy", or the model is good.

- We can only compare different models on the same corpus.

- Should we measure on training data or held-out data? Why?

Sparse data, again

Suppose now we build a *trigram* model from Moby Dick and evaluate the same sentence.

- But I spent three never occurs, so $P_{MLE}(\text{three} \mid \text{I spent}) = 0$
- which means the cross-entropy is infinite.
- Basically right: our model says I spent three should never occur, so our model is infinitely wrong/surprised when it does!
- Even with a unigram model, we will run into words we never saw before. So even with short N-grams, we need better ways to estimate probabilities from sparse data.

Smoothing

- The flaw of MLE: it estimates probabilities that make the training data maximally probable, by making everything else (unseen data) minimally probable.
- **Smoothing** methods address the problem by stealing probability mass from seen events and reallocating it to unseen events.
- Lots of different methods, based on different kinds of assumptions. We will discuss just a few.

Add-One (Laplace) Smoothing

- Just pretend we saw everything one more time than we did.

$$P_{\mathrm{ML}}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}$$

$$\Rightarrow \qquad P_{+1}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i) + 1}{C(w_{i-2}, w_{i-1})} \qquad ?$$

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- NO! Sum over possible w_i (in vocabulary V) must equal 1:

$$\sum_{w_i \in V} P(w_i|w_{i-2},w_{i-1}) = 1$$

- If increasing the numerator, must change denominator too.

Add-one Smoothing: normalization

- We want:
$$\sum_{w_i \in V} \frac{C(w_{i-2}, w_{i-1}, w_i) + 1}{C(w_{i-2}, w_{i-1}) + x} = 1$$

- Solve for x:

$$\begin{aligned}
\sum_{w_i \in V} (C(w_{i-2}, w_{i-1}, w_i) + 1) &= C(w_{i-2}, w_{i-1}) + x \\
\sum_{w_i \in V} C(w_{i-2}, w_{i-1}, w_i) + \sum_{w_i \in V} 1 &= C(w_{i-2}, w_{i-1}) + x \\
C(w_{i-2}, w_{i-1}) + v &= C(w_{i-2}, w_{i-1}) + x \\
v &= x
\end{aligned}$$

where $v =$ vocabulary size.

Add-one example (1)

- *Moby Dick* has one trigram that begins I spent (it's I spent in) and the vocabulary size is 17231.

- Comparison of MLE vs Add-one probability estimates:

	MLE	+1
$\hat{P}(\text{three} \mid \text{I spent})$	0	0.00006
$\hat{P}(\text{in} \mid \text{I spent})$	1	0.0001

- $\hat{P}(\text{in}|\text{I spent})$ seems very low, especially since in is a very common word. But can we find better evidence that this method is flawed?

Add-one example (2)

- Suppose we have a more common bigram w_1, w_2 that occurs 100 times, 10 of which are followed by w_3.

	MLE	+1
$\hat{P}(w_3 \mid w_1, w_2)$	$\frac{10}{100}$	$\frac{11}{17331}$
		≈ 0.0006

- Shows that the very large vocabulary size makes add-one smoothing steal *way* too much from seen events.

- In fact, MLE is pretty good for frequent events, so we shouldn't want to change these much.

Add-α (Lidstone) Smoothing

- We can improve things by adding $\alpha < 1$.

$$P_{+\alpha}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i) + \alpha}{C(w_{i-1}) + \alpha v}$$

- Like Laplace, assumes we know the vocabulary size in advance.
- But if we don't, can just add a single "unknown" (UNK) item to the vocabulary, and use this for all unknown words during testing.
- Then: how to choose α?

Optimizing α (and other model choices)

- Use a three-way data split: **training** set (80-90%), **held-out** (or **development**) set (5-10%), and **test** set (5-10%)
  - Train model (estimate probabilities) on training set with different values of α
  - Choose the α that minimizes cross-entropy on development set
  - Report final results on test set.
- More generally, use dev set for evaluating different models, debugging, and optimizing choices. Test set simulates deployment, use it only once!
- Avoids overfitting to the training set and even to the test set.

Summary

- We can measure the relative goodness of LMs on the same corpus using cross-entropy: how well does the model predict the next word?
- We need smoothing to deal with unseen N-grams.
- Add-1 and Add-α are simple, but not very good.
- We'll see even better smoothing methods next time.