Classification: Naïve Bayes

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(slides adapted from Chris Dyer, Noah Smith, Sharon Goldwater, et al.)
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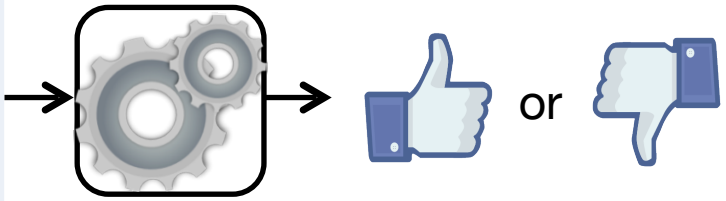
Sentiment Analysis

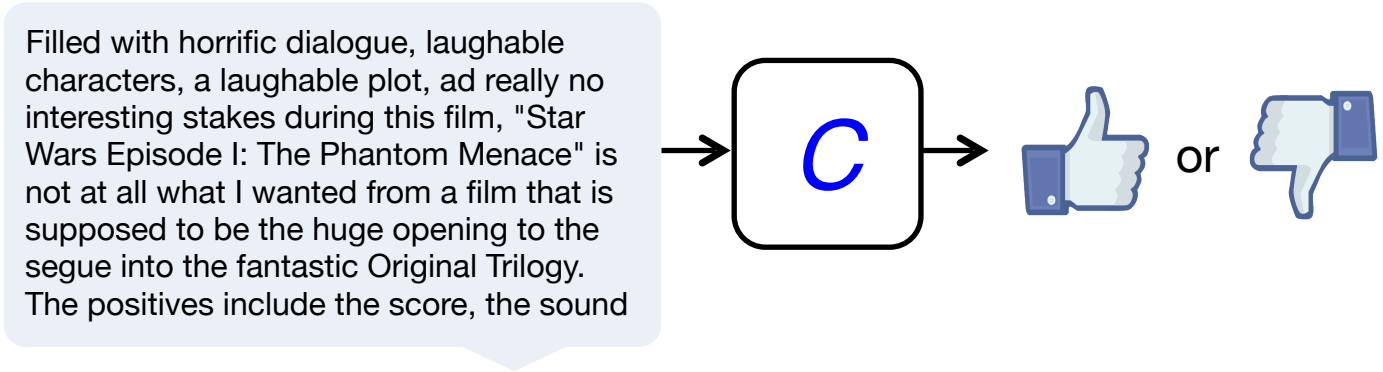
- Recall the task:

Filled with horrific dialogue, laughable characters, a laughable plot, ad really no interesting stakes during this film, "Star Wars Episode I: The Phantom Menace" is not at all what I wanted from a film that is supposed to be the huge opening to the segue into the fantastic Original Trilogy. The positives include the score, the sound



Sentiment Analysis

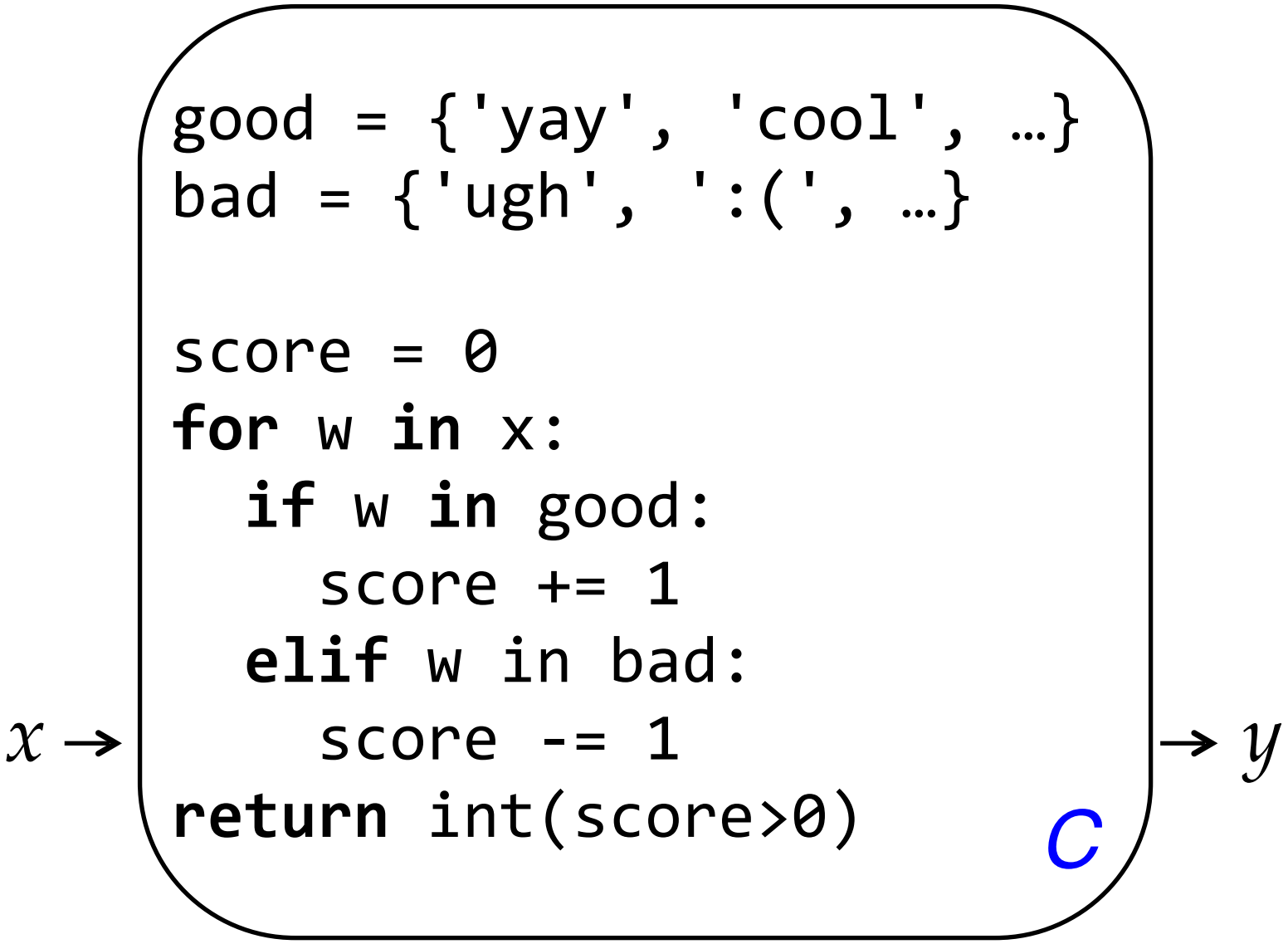
- Recall the task:



- This is a **classification** task: we have open-ended text as *input* and a fixed set of discrete classes as *output*.

- By convention, the input/observed information is denoted x, and the output/predicted information is y.

A Rule-based Classifier

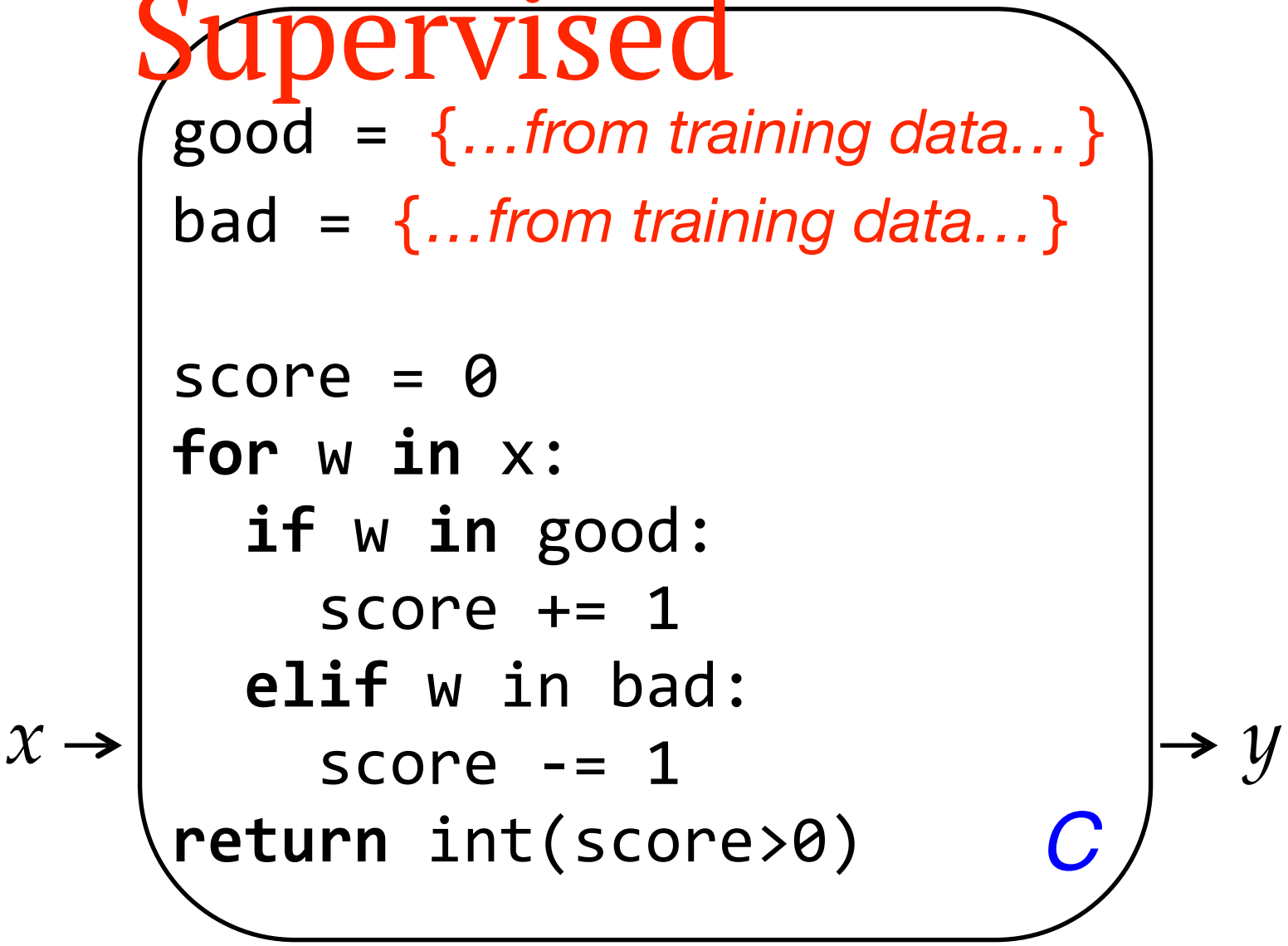


Supervised Classification

- We can probably do better with data
  - Our intuitions about word sentiment aren't perfect
- **Supervised** = generalizations are **learned** from **labeled** data
  - So, we need a **training** corpus of reviews with gold (correct) sentiment labels
  - And a learning algorithm
- This course: **inductive** learning algorithms—collect statistics from training corpus, but the resulting classifier does not rely on the training corpus itself

A ~~Rule-based~~ Classifier

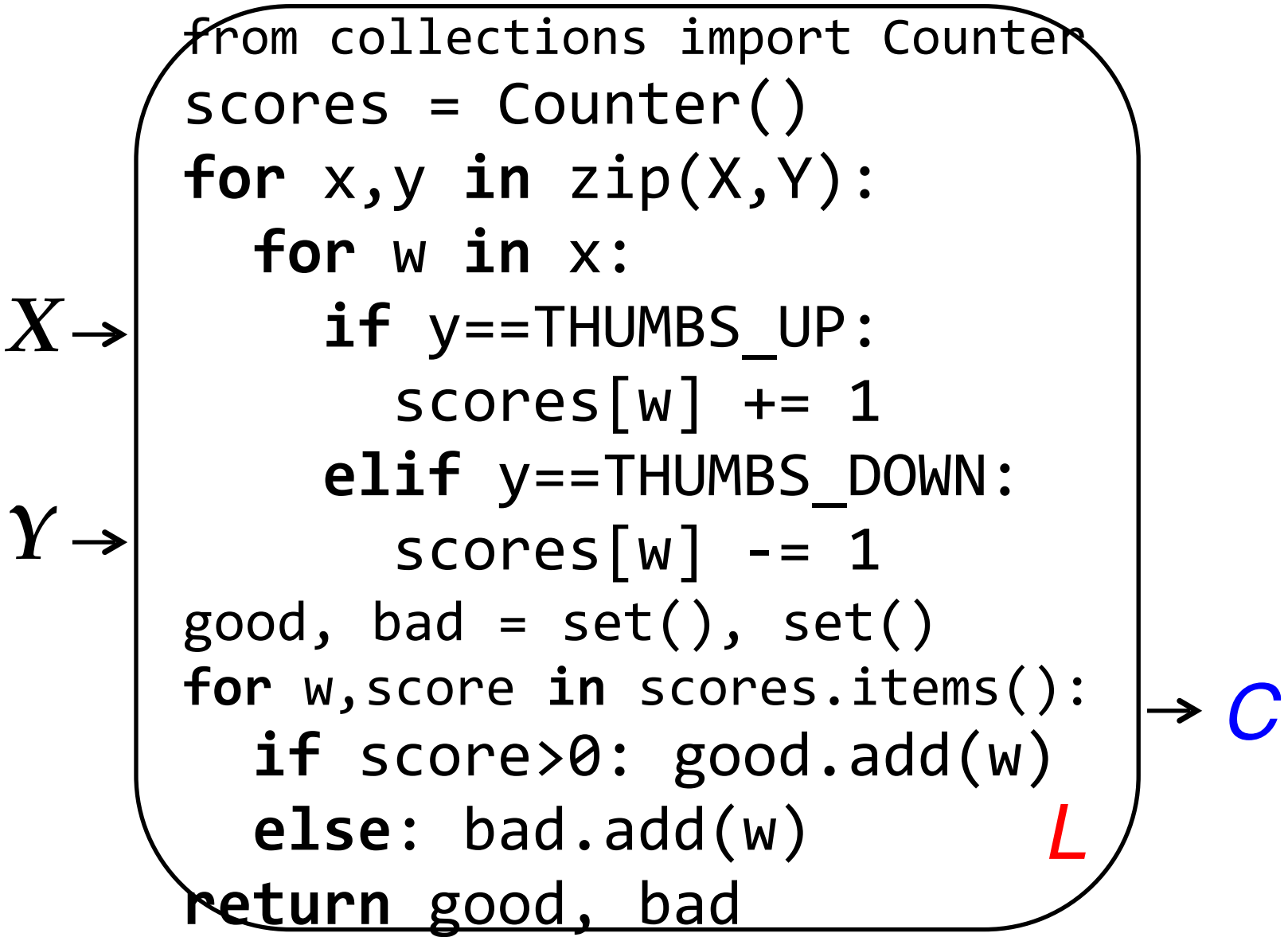
Supervised



Notation

- Training examples: $\boldsymbol{X} = (x_1, x_2, \ldots, x_N)$
- Their categories: $\boldsymbol{Y} = (y_1, y_2, \ldots, y_N)$
- A **classifier** *C* seeks to map x_i to y_i: $x \rightarrow$ C $\rightarrow y$
- A **learner** *L* infers *C* from $(\boldsymbol{X}, \boldsymbol{Y})$: $\boldsymbol{X} \rightarrow$, $\boldsymbol{Y} \rightarrow$ L $\rightarrow$ C

Counting as Learning



Limitations

- Our classifier doesn't know that:
  - Some words are more strongly indicative of sentiment than others
  - The data may skew positive or negative (e.g., more or longer positive reviews than negative)
  - Infrequent words may occur only in the positive examples or only in the negative examples by accident
- Instead of raw counts, we can use a **probabilistic model**

Review Questions: Conditional Probability

1. If *p* is a probability mass function, which is true by the definition of conditional probability:
   p(x | y, z) =

   a. p(x)/p(y,z)

   b. p(y)p(z)/p(x,y,z)

   c. p(x,y,z)/p(y,z)

   d. p(x)p(x|y)p(x|z)

Review Questions: Conditional Probability

2. Which is/are guaranteed to be true?

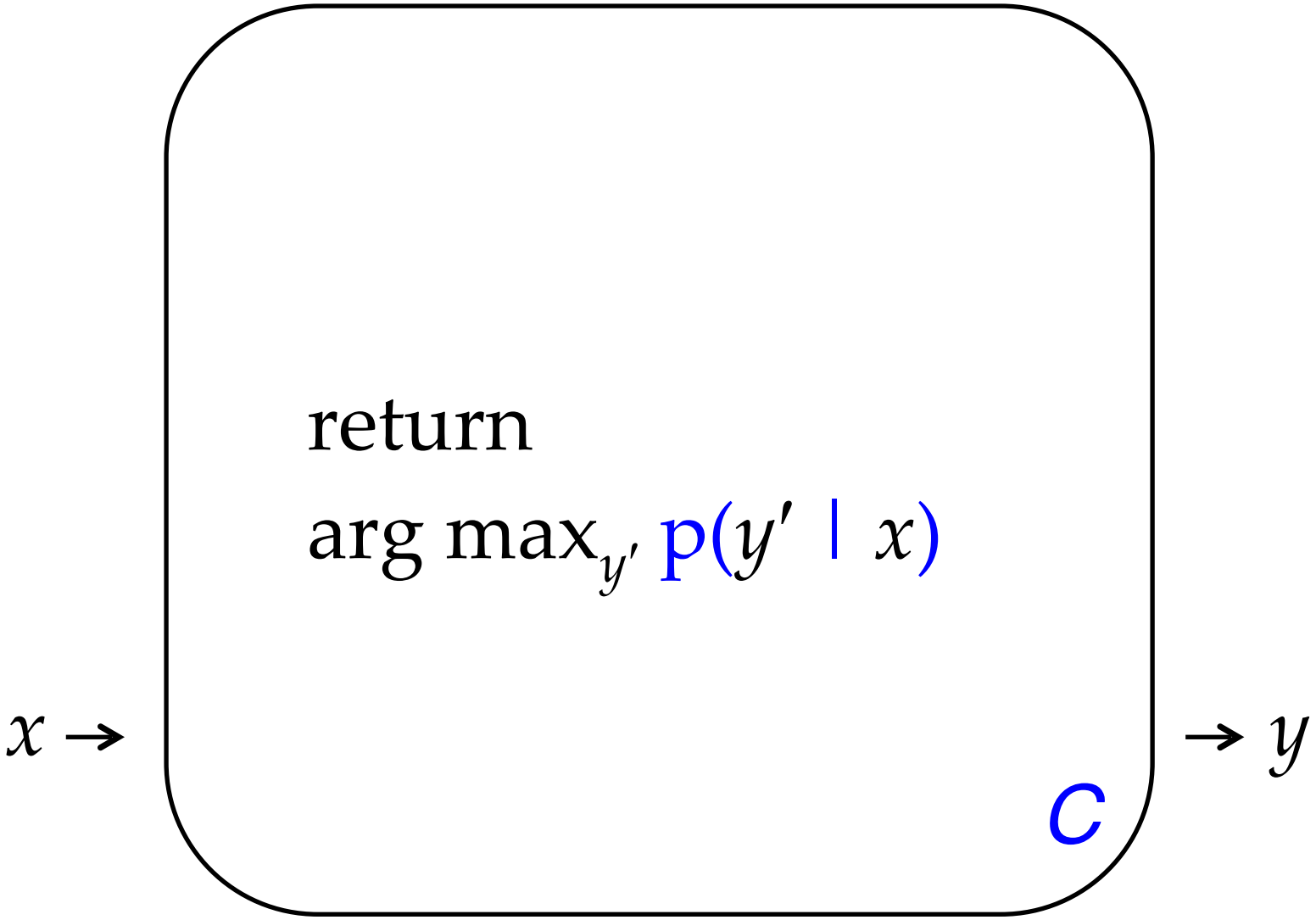
   a. $\forall y\ \forall z,\ \Sigma_x\ p(x \mid y, z) = 1$

   b. $\forall x,\ \Sigma_y\ \Sigma_z\ p(x \mid y, z) = 1$

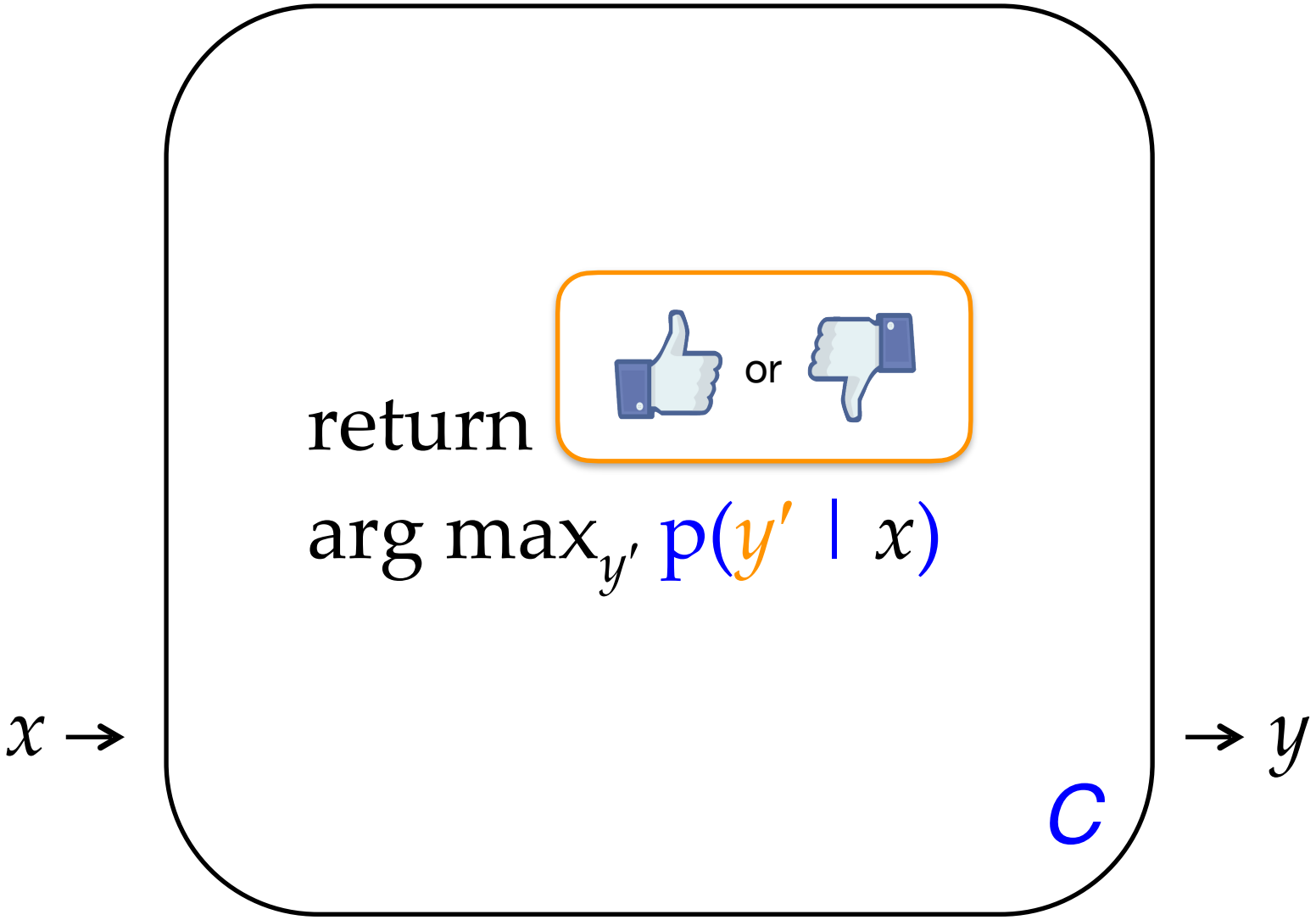
   c. $\Sigma_x\ p(x) = 1$

   d. $\forall y\ \forall z,\ \Sigma_x\ p(x)p(y|x)p(z|x,y) = 1$

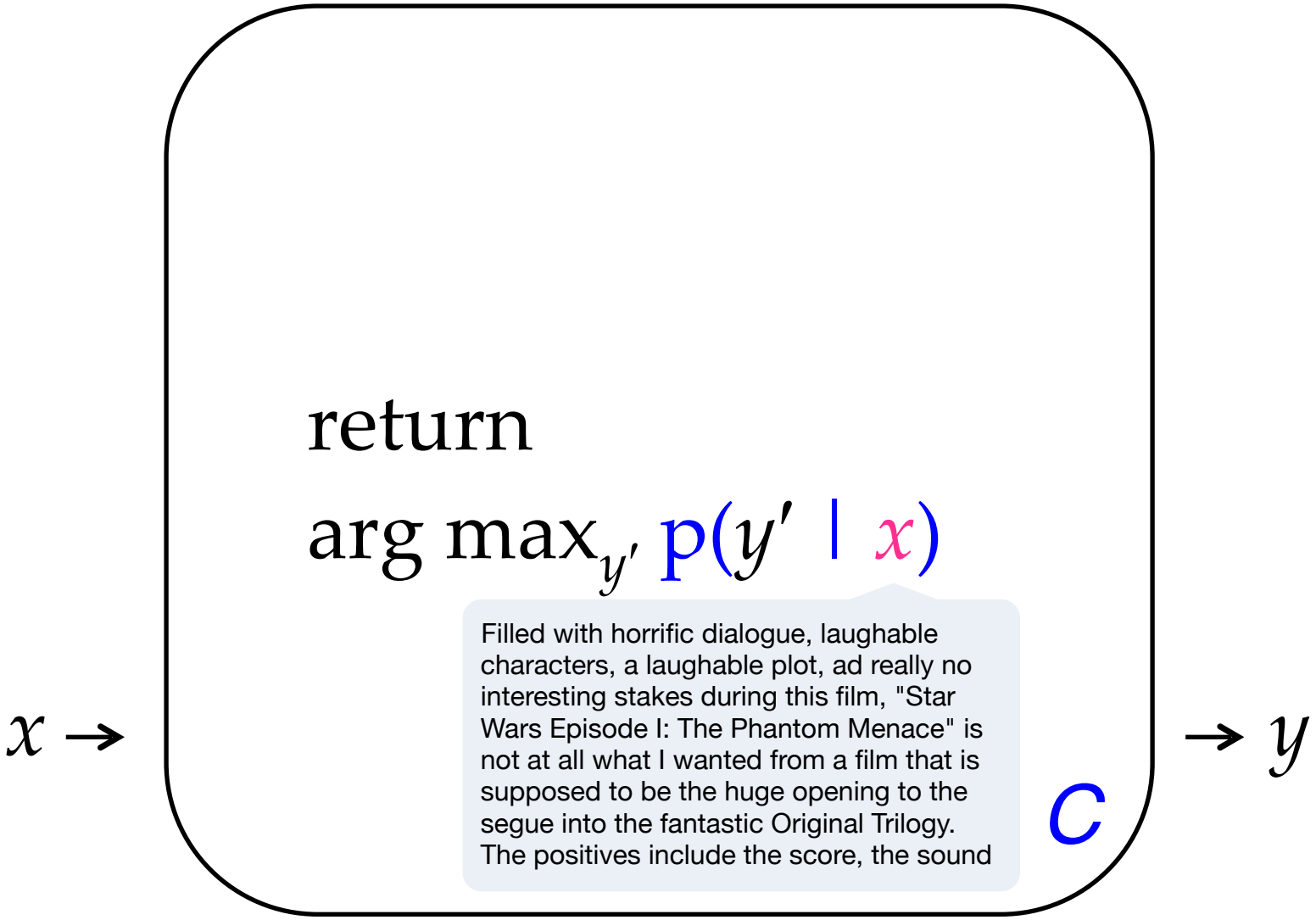
Probabilistic Classifiers



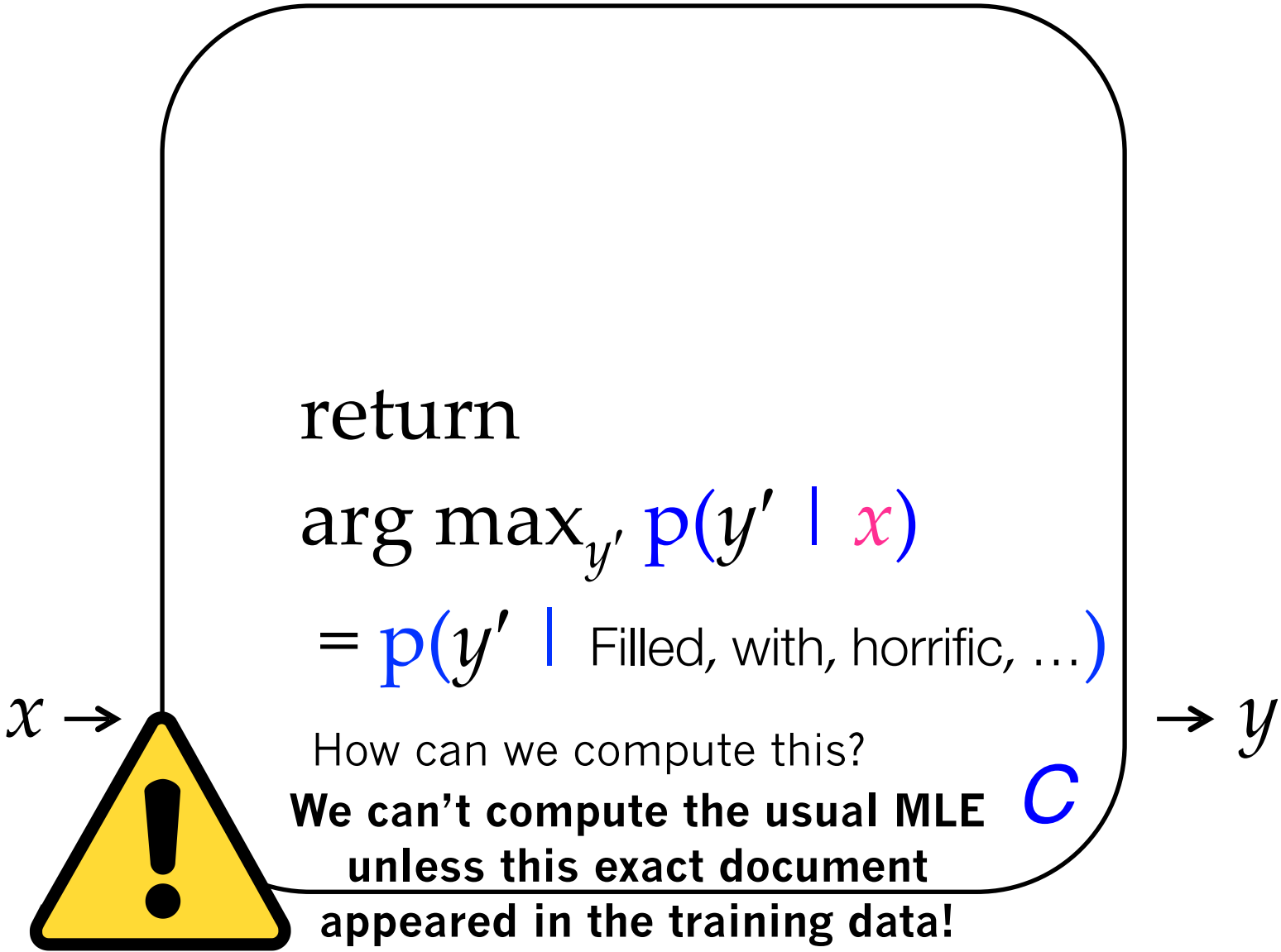
Probabilistic Classifiers



Probabilistic Classifiers



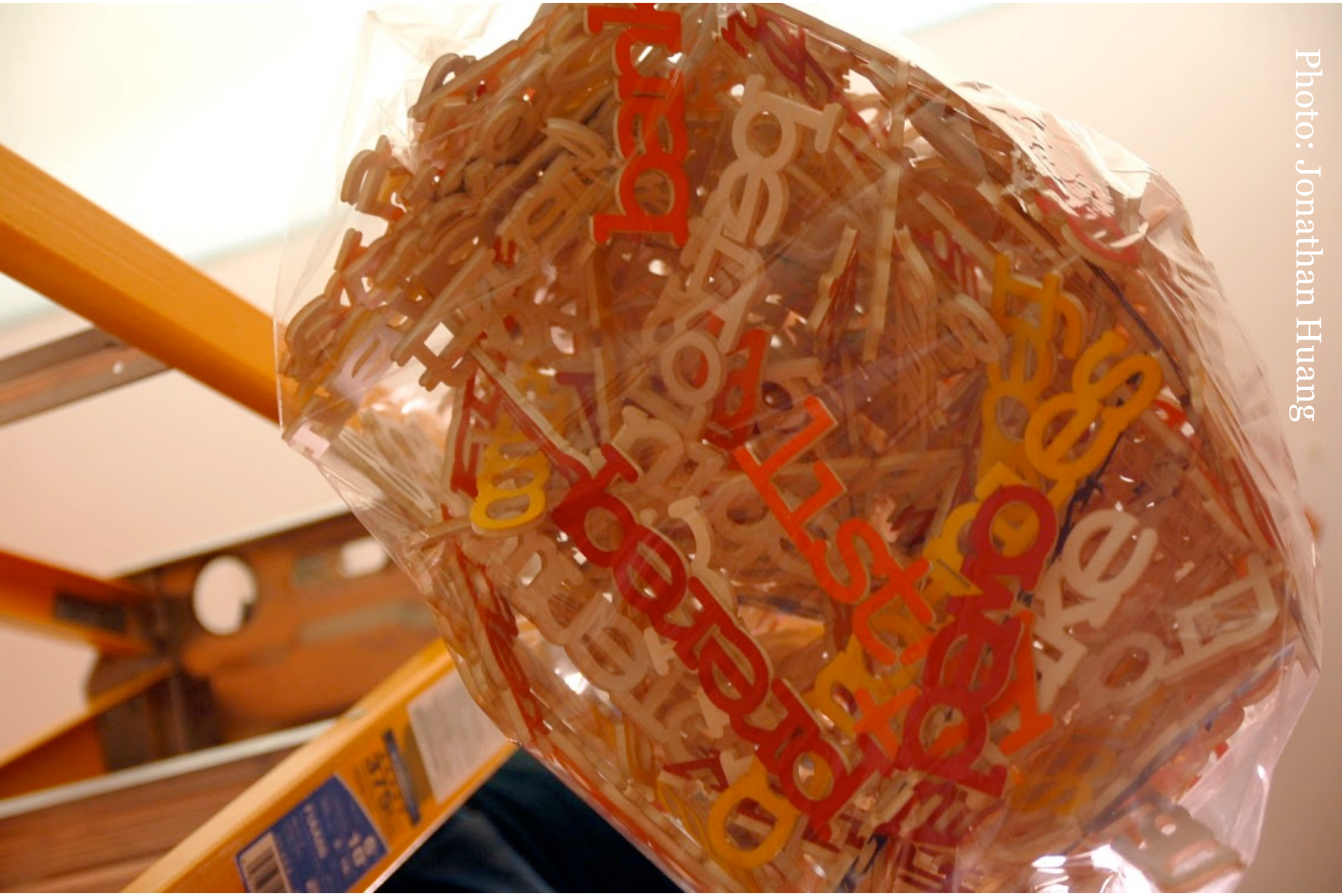
Probabilistic Classifiers



A probabilistic model that generalizes

- Instead of estimating p(*y′* | Filled, with, horrific, …) directly, we make two **modeling assumptions**:

  1. The **Bag of Words (BoW) assumption:** Assume the order of the words in the document is irrelevant to the task. I.e., stipulate that
     p(*y′* | Filled, with, horrific) = p(*y′* | Filled, horrific, with)



Art installation in CMU's Machine Learning Department

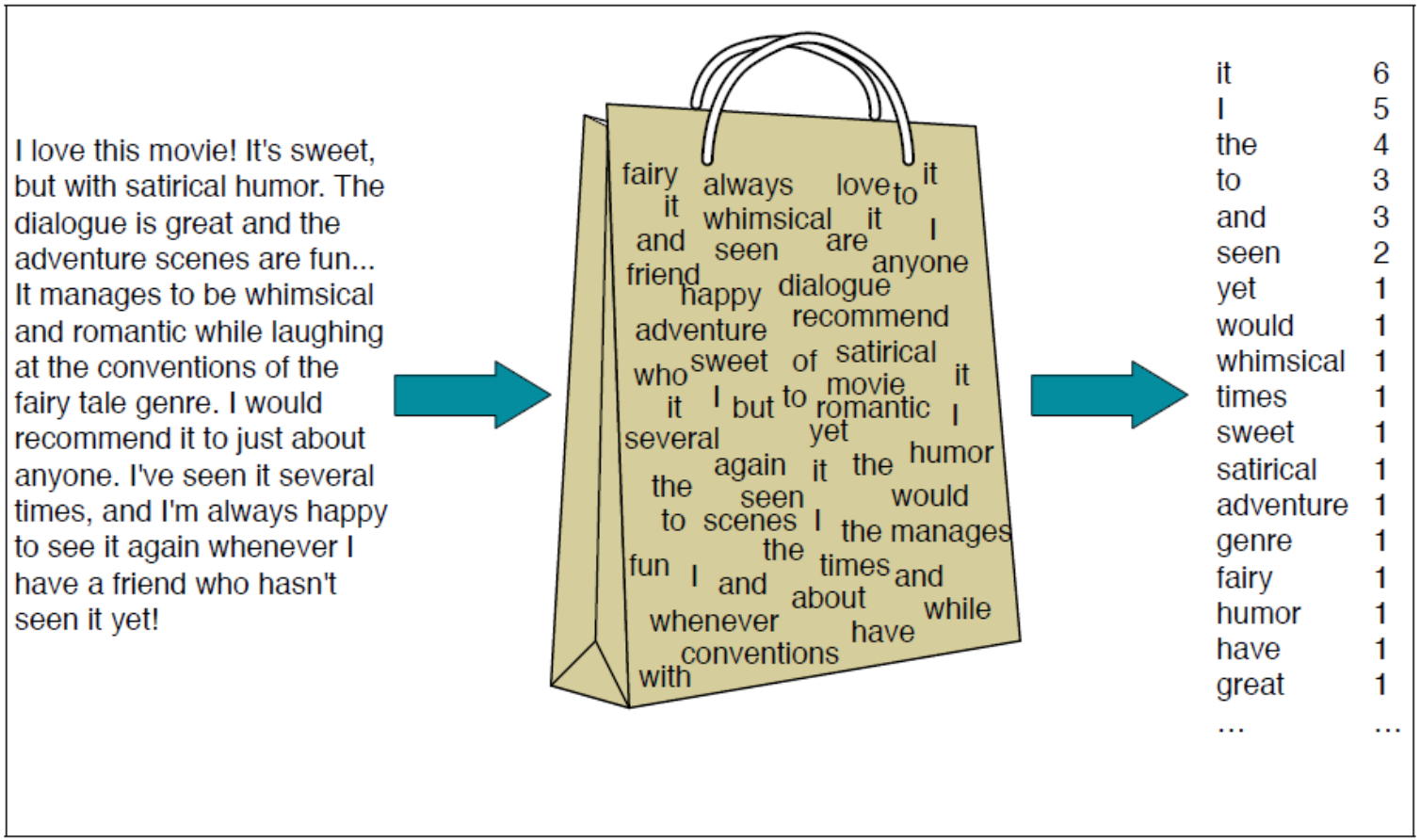


Figure 7.1 Intuition of the multinomial naive Bayes classifier applied to a movie review. The position of the words is ignored (the *bag of words* assumption) and we make use of the frequency of each word.

Figure from J&M 3rd ed. draft, sec 7.1

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     p(*y′* | Filled, with, horrific) = p(*y′* | Filled, horrific, with)

     So called because a **bag** or **multiset** is a data structure that stores counts of elements, but not their order.

A probabilistic model that generalizes

- The BoW assumption isn't enough, though, unless documents with all the same words occurred in the training data. Hence:

2. The **naïve Bayes assumption:** Assume the words are **independent** conditioned on the class *y′*

$$p(\text{Filled, with, horrific} \mid y') = p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$$

Hang on, we actually wanted:

$$p(y' \mid \text{Filled, with, horrific})$$

How to reverse the order?

Bayes' Rule

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

Prove it!

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

multiply both sides by p(A)

$$p(A) \times p(B \mid A) = p(B) \times p(A \mid B)$$

Chain Rule

$$p(A, B) = p(B, A)$$

...which is true by definition of joint probability

Bayes' Rule

$$p(B \mid A) = \frac{p(B) \times p(A \mid B)}{p(A)}$$

$$p(B \mid A) \propto p(B) \times p(A \mid B)$$

posterior **prior** **likelihood**

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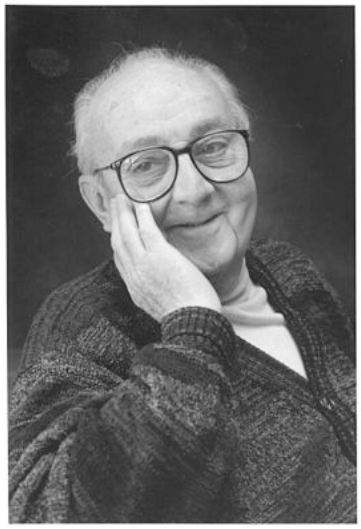
$$p(y' \mid \text{Filled, with, horrific}) \propto p(y') \times p(\text{Filled, with, horrific} \mid y') = p(y') \times p(\text{Filled} \mid y') \times p(\text{with} \mid y') \times p(\text{horrific} \mid y')$$

Is this a good model?

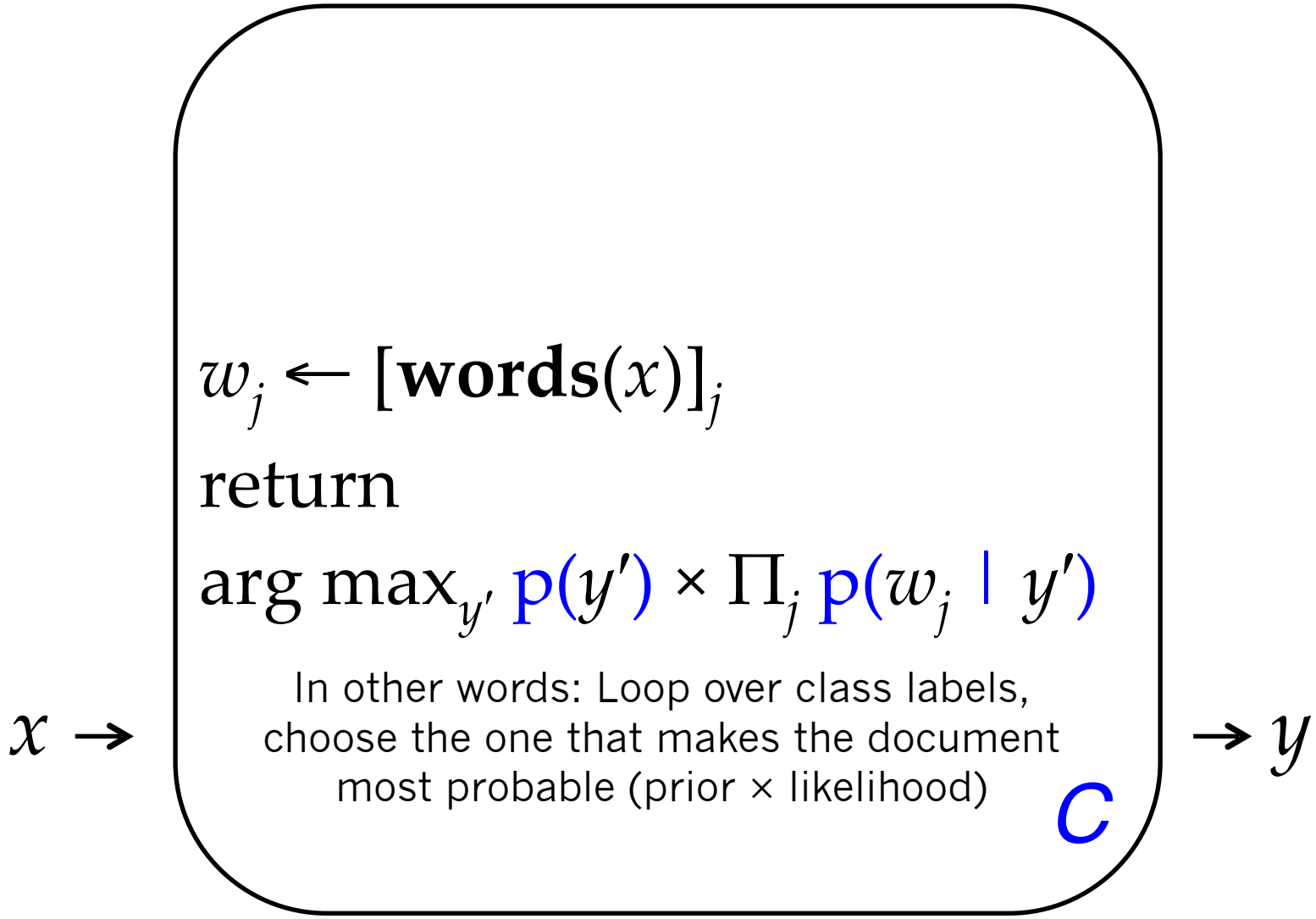
- What is wrong with these assumptions?

Is this a good model?

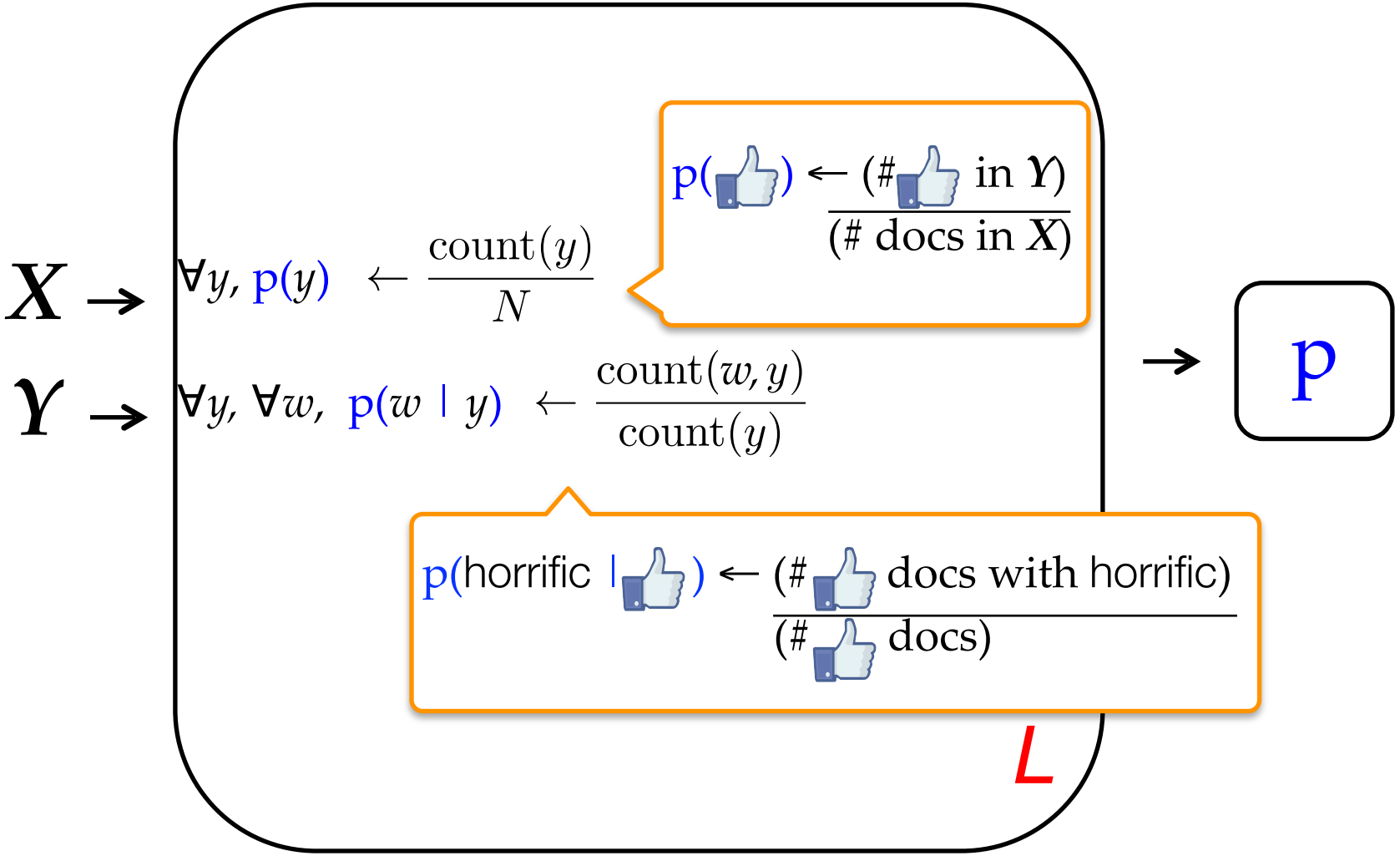
- George Box, statistician: "essentially, **all models are wrong, but some are useful**")
- It turns out that naïve Bayes + BoW works pretty well for many text classification tasks, like spam detection.



Naïve Bayes Classifier



Naïve Bayes Learner



Parameters

- Each probability (or other value) that is **learned** and used by the classifier is called a **parameter**
  - E.g., a single probability in a distribution
- Naïve Bayes has two kinds of distributions:
  - the class prior distribution, **p(y)**
  - the likelihood distribution, **p(w | y)**
- So how many parameters total, if there are K classes and V words in the training data?

Smoothing p(*w* | *y*)

p(horrific | 👍) ← (# 👍 docs with horrific) / (# 👍 docs)

- What if we encounter the word distraught in a test document, but it has never been seen in training?
  - Can't estimate p(distraught | 👍) or p(distraught | 👎): numerator will be 0
  - Because the word probabilities are multiplied together for each document, the probability of the whole document will be 0

Smoothing p(*w* | *y*)

$$p(\text{horrific} \mid \text{👍}) \leftarrow \frac{(\#\ \text{👍 docs with horrific}) + 1}{(\#\ \text{👍 docs}) + V + 1}$$

$$p(\text{OOV} \mid \text{👍}) \leftarrow \frac{1}{(\#\ \text{👍 docs}) + V + 1}$$

V is the size of the vocabulary of the training corpus

- **Smoothing** techniques adjust probabilities to avoid **overfitting** to the training data
  - Above: **Laplace (add-1) smoothing**
  - OOV (out-of-vocabulary/unseen) words now have small probability, which decreases the model's confidence in the prediction without ignoring the other words
  - Probability of each seen word is reduced slightly to save probability mass for unseen words

Smoothing p(*w* | *y*)

$$p(\text{horrific} \mid 👍) \leftarrow \frac{(\#\ 👍 \text{ docs with horrific}) + 1}{(\#\ 👍 \text{ docs}) + V + 1}$$

$$p(\text{OOV} \mid 👍) \leftarrow \frac{1}{(\#\ 👍 \text{ docs}) + V + 1}$$

V is the size of the vocabulary of the training corpus

- **Laplace (add-1) smoothing**, above, uses a **pseudo-count** of 1, which is kind of arbitrary.
  - For some datasets, it's overkill—better to smooth less.
  - **Lidstone (add-α) smoothing**: **tune** the amount of smoothing on **development** data:

$$p(\text{horrific} \mid 👍) \leftarrow \frac{(\#\ 👍 \text{ docs with horrific}) + \alpha}{(\#\ 👍 \text{ docs}) + \alpha(V + 1)}$$

$$p(\text{OOV} \mid 👍) \leftarrow \frac{\alpha}{(\#\ 👍 \text{ docs}) + \alpha(V+1)}$$

Naïve Bayes Classifier

$x \rightarrow$

$w_j \leftarrow [\mathbf{words}(x)]_j$

return

$\arg\max_{y'} \mathrm{p}(y') \times \prod_j \mathrm{p}(w_j \mid y')$

In other words: Loop over class labels, choose the one that makes the document most probable (prior × likelihood)

C

Can get very small

$\rightarrow y$

Avoiding Underflow

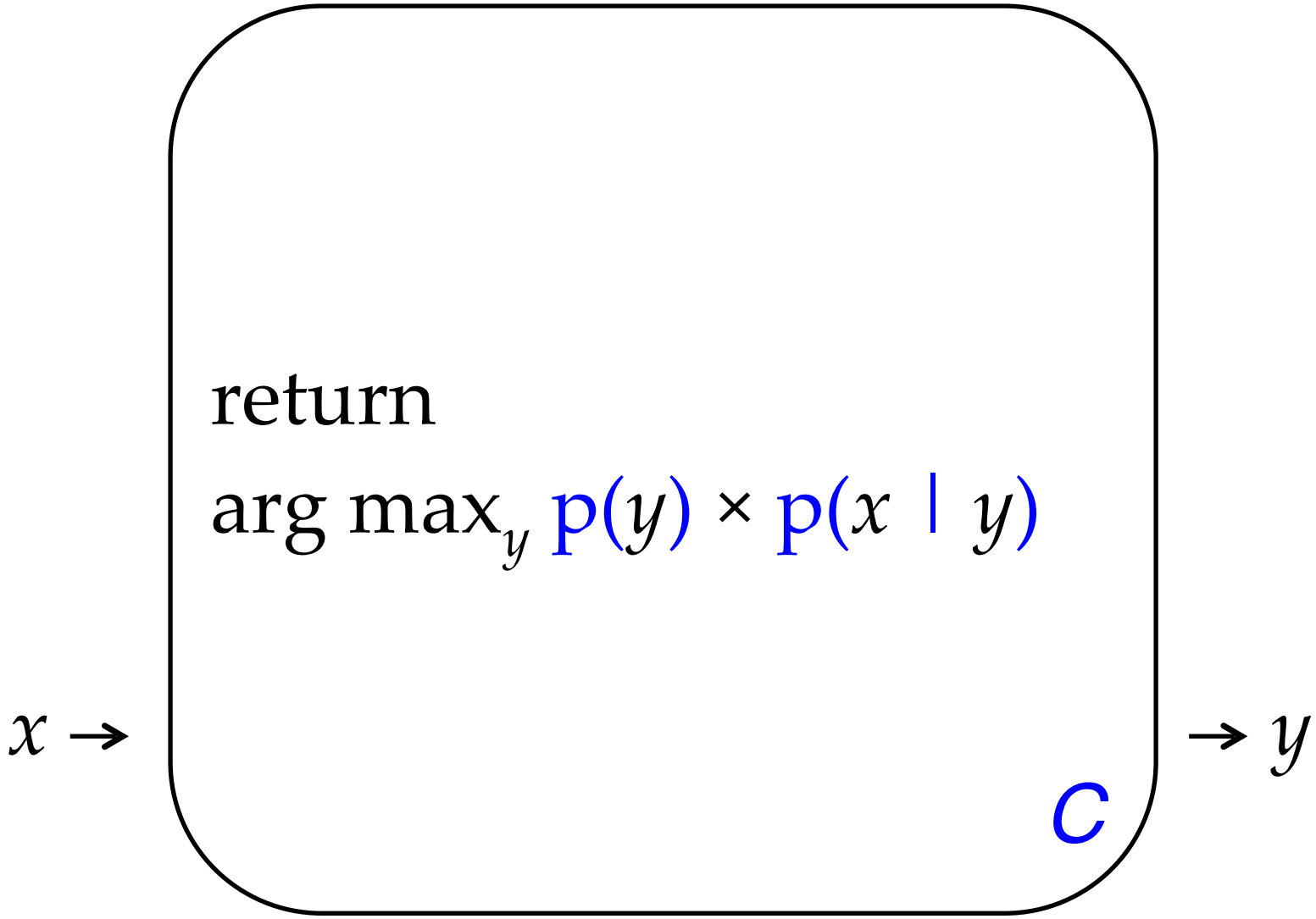
- Multiplying 2 very small floating point numbers can yield a number that is too small for the computer to represent. This is called **underflow**.
- In implementing probabilistic models, we use **log probabilities** to get around this.
  - Instead of storing $p(\bullet)$, store $\log p(\bullet)$
  - $p(\bullet) \times p'(\bullet) \rightarrow \log p(\bullet) + \log p'(\bullet)$
  - $p(\bullet) + p'(\bullet) \rightarrow$ `numpy.logaddexp`$(\log p(\bullet), \log p'(\bullet))$

Noisy Channel Model

What proportion of emails are expected to be spam vs. not spam?

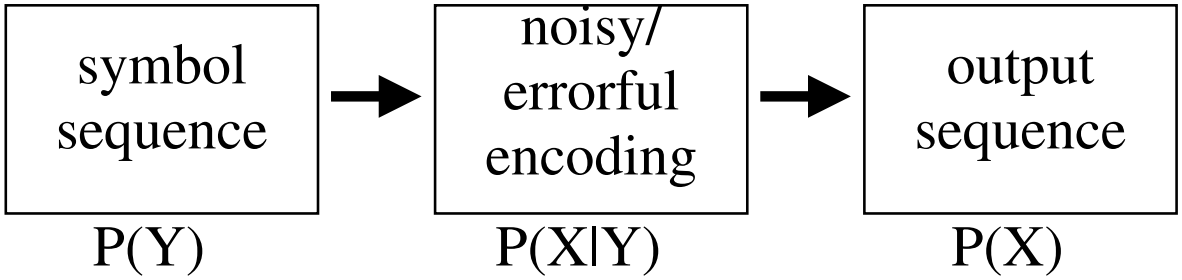
What proportion of product reviews are expected to get 1,2,3,4,5 stars?

Noisy Channel Classifiers



Noisy Channel Model

- We imagine that someone tries to communicate a sequence to us, but noise is introduced. We only see the output sequence.



Application	Y	X
Speech recognition	spoken words	acoustic signal
Machine translation	words in L_1	words in L_2
Spelling correction	intended words	typed words

NC Example: Spelling Correction

- $P(Y)$: Distribution over the words the user intended to type. A language model!
- $P(X|Y)$: Distribution describing what user is **likely** to type, given what they **meant**. Could incorporate information about common spelling errors, key positions, etc. Call it a **noise model**.
- $P(X)$: Resulting distribution over what we actually see.
- Given some particular observation x (say, effert), we want to recover the most probable y that was intended.

Conclusions

- We have seen how labeled **training data** and **supervised learning** can produce a better-informed classifier
  - Classifier takes an *input* (such as a text document) and predicts an *output* (such as a class label)
  - Learner takes *training data* and produces (statistics necessary for) the classifier

Conclusions

- Because most pieces of text are unique, it's not very practical to assume the one being classified has occurred in the training data
  - We need to make **modeling assumptions** that help the learner to **generalize** to unseen inputs
- The **naïve Bayes** model + **bag-of-words** assumption are a simple, fast probabilistic approach to text classification
  - Works well for many tasks, despite being a ~~dumb~~ naïve model of language: We know that
    - *good, not as bad as expected ≠ bad, not as good as expected*
    - p(Star Wars | 👍) ≠ p(Star | 👍) × p(Wars | 👍)

Conclusions

- In practice, we need **smoothing** to avoid assuming that everything that might come up at test time is in the training data
- Implementation trick: use **log probabilities** to avoid underflow